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The Institute of Actuaries.

MESSENGER PRIZE ESSAY

ON

FRIENDLY SOCIETIES.

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BY

GEORGE F. HARDY,

FELLOW OF THE INSTITUTE OF ACTUARIES.

Published by the Authority of The Institute of Actuaries.

LONDON:

CHARLES AND EDWIN LAYTON,

56, FARRINGDON STREET, E.C.

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GENERAL

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SYLLABUS OF THE MESSENGER PRIZES, 1886.

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Subject—Friendly Societies.

The Council of The Institute of Actuaries, in an announcement contained in the January 1887 number of the *Journal*, stated that they had resolved to offer a Prize of £50 to the Author of the best Essay upon this subject, and a further sum of £25 to the Author of the second best Essay, or divided between two or more Essayists, if and as the Adjudicators might recommend.

The Council, without restricting Candidates in the treatment of the subject, thought it desirable to make the following suggestions as to what the Essay should include:

"1. A brief historical sketch of the Friendly Society system in this country, including a short account of the salient features of the legislation on the subject, particularly that of recent years. Reference is invited also to the Friendly Societies, and to the legislation relating thereto, in other countries.

"2. A description of the various classes of Friendly Societies at the present time, including the affiliated orders, such as the Manchester Unity of Oddfellows, the Ancient Order of Foresters, &c. It is desirable that the description should include a classification of societies in respect of the nature of the benefits assured to the members, and the manner in which those benefits are paid for—whether by fixed periodic contributions, by levies to meet current claims, by levies of fixed amount on the happening of certain events, &c.—including also the cases where societies are subsidized from extraneous sources.

"3. An enquiry into the applicability of actuarial principles and data to the various classes of societies, both in respect to the benefits assured and the methods in use with regard to payments for those benefits.

"In this section should also be included: (*a*) an investigation into the proper methods of keeping accurate records of the experience

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of societies, and of obtaining therefrom the necessary data for actuarial purposes; (*b*) a critical account of well-known existing data, and an enquiry as to how far they may be made use of for the purposes of particular societies; (*c*) an exact enquiry into the effect of secessions in the various classes of societies, and the manner in which, and extent to which, secessions should be introduced into valuations of their contracts—and in dealing with this question, the treatment of negative values should receive careful consideration; (*d*) an examination of the relative effect of rates of mortality and rates of sick pay in determining the values of sick-pay benefits—in other words, how far the rates of sick-pay benefit will be interfered with by a light or heavy mortality; (*e*) an enquiry as to the number under risk in various classes of societies necessary to render probable a steady experience, particular attention being given to the difference (if any) in this respect between societies granting death benefits only and those granting sick-pay or accident benefits, with or without death benefits. And, where practicable, statistics should be furnished bearing upon the various points under discussion.

"The Essays may, *inter alia*, indicate such changes in the existing practice of societies, or in the existing legislation and accompanying executive arrangements, as seem to the writer desirable—and in dealing with this part of the subject a comparison is invited between the legislation relating to Friendly Societies and that affecting ordinary life insurance companies."

"CONDITIONS OF THE COMPETITION.

"1. That the Competition be open to all Members of the Institute, with the exception of past or present members of Council.

"2. That the Essays be sent in to the President of the Institute before or on 31 December 1887.

"(*If Essays are written in a foreign language they should be accompanied by a literal English translation.*)

"3. That the names of the Competitors be sent in under seal, with a Motto corresponding to one to be prefixed to the Essay; the Motto and Essay not to be in the handwriting of the Competitor.

"4. That the Essay or Essays to which a Prize may be awarded be the property of the Institute.

"5. That a Prize or Prizes be only awarded if the Adjudicators shall consider an Essay or Essays to be worthy of the distinction.

"6. That unsuccessful Essays be returned, on application at the Institute, with the corresponding envelopes unopened."

[*Printed by the Authority of The Institute of Actuaries.*]

MESSENGER PRIZE ESSAY

ON

FRIENDLY SOCIETIES.

BY

GEORGE F. HARDY,

FELLOW OF THE INSTITUTE OF ACTUARIES.

Historical Sketch.

Early Times.

In considering the origin and development of the friendly society system, it is necessary to recognize the twofold purpose that these institutions have fulfilled. In the view of the actuary they are merely associations for the mutual assurance of certain benefits, generally in the case of the sickness or death of any of the members. This, however, represents only one side, and that by far the most recent, of the work of these institutions. A not less prominent and much older feature is to be found in the opportunities for social intercourse, companionship, and mutual help, which such associations naturally afford—wants common to man in every state of society, and as old as civilization itself. Hence, from the earliest times, small communities have existed, drawn together by the common interests, tastes, or pursuits of the members, such communities being

originally purely social in character, subsequently charitable and mutually protective, and only in recent times financial.

Institutions very closely agreeing, in many of their characteristics, with the friendly societies of the present day, date from a very remote period. Mr. Tompkins, in his essay on the *Friendly Societies of Antiquity*, has shown that they existed among both Greek and Roman communities before the Christian era, and that, besides meeting for religious observances, they possessed funds which were employed to provide funerals and to assist members in needy circumstances, although there is no evidence of the grant of benefits in cases of sickness. The connecting links between these early societies and those of modern times are difficult to trace; nor is it necessary to suppose that the latter are merely a development of the former. The history of the friendly society movement in our own country has been very ably sketched by Dr. Hasbach, of Leipsic (*Das Englishe Arbeiterversicherungswesen*—Leipsic, 1883), of whose work I have availed myself to some extent in the following short sketch. The Rev. J. Frome Wilkinson's work on the same subject contains also much useful information, more especially in relation to the rise of the affiliated orders.

Prior to the 13th century, aid was generally dispensed to the necessitous by the Church. Being the central and only receiver of contributions for this purpose, and the sole dispenser of relief, its charitable functions covered the same kind of wants that our friendly societies deal with at the present time. These functions were to feed and clothe the destitute, to visit the sick, to house the homeless, to liberate prisoners, and to bury the dead—in its own words, *vito, poto, cibo, redimo, tego, colligo, condo.* Such duties did not remain merely voluntary. Grants, tithes, bequests, &c., gradually increasing the property of the Church, imposed at the same time the tacitly understood obligation to satisfy the wants of the poor, and at a later period we find the obligatory nature of the duties of the Church to the poor emphasized and confirmed by various legislative enactments, such as the Act 9 of Edward I, and Act 15 of Richard II, and others.

13th and 14th Centuries. The Guilds.

In the 13th and 14th centuries, a great social change took place. Agricultural serfdom was gradually breaking up, industries began to develop, and a general tendency was manifested by society to forsake the socialistic order of things. As regards the relief of the poor, the first effect of the change was the taking over of part

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of the functions hitherto performed by the Church by smaller communities of persons, each community, or guild, as they were called, having a more or less distinct and well-defined object. These guilds undertook, though usually as an object of secondary importance, the distribution of relief amongst their members, constituting them the first friendly societies in this country of which we have any clear information. They appear to have flourished up to the time of the Reformation, and most of them had, according to Mr. Toulmin Smith, particular provisions for the optional relief, by way of charity, of those members of the guild who were in want. The Guild of St. Katherine, Aldersgate (and many others), contained stipulations in its statutes decreeing support to brethren in case of poverty, old age, damage by fire or water, by thieves or sickness. Others undertook special tasks, apart from their general charity. The Guild of Garlekhith of St. Leonard's, Lynn, of the Smiths of Chesterfield, and others, gave relief in case of any of their members being cast into prison. The Guild of the Palmers provided dowries for women, and others, too numerous to mention, gave relief in a vast variety of cases. The following paragraph in the statutes of the Guild of Smiths of Chesterfield is of great interest:—"When a brother is ill and needs relief, then he shall "receive half a dime daily; when brethren fall into poverty, then "they shall go singly, on certain days, into the houses of their "brethren, where each one shall be received civilly, and he shall "obtain whatsoever he may need in the shape of food and clothing, "as if he were the master of the house himself; and he shall also "receive half a dime, like those who are ill, and then he shall go "his way in peace." It is to these guilds that we must look, rather than the protective trade guilds which then existed, for the progenitors of our modern friendly societies. As to their financial basis, we find that in some cases the contributions were equal; in others they varied according to the wealth of the member, each contributing as much as he could to the common fund, not distinctly for his own benefit, but for the benefit of the members generally, or the community at large.

Period of the Reformation.

The period of the Reformation was another era of change in the direction of self-assertion and self-help, and the general tendency affected the guilds quite as much, perhaps, as the change from Catholicism to Protestantism. In the 15th century they began to decay. Corruption and abuses were not uncommon amongst them, as in the convents and other religious

foundations. This state of things culminated in the 16th century in the passing of Acts 27, 31, and 37 of Henry VIII, and 1 of Edward VI, which enabled the king to confiscate the property of both guilds and monasteries; and although Parliament laboriously stated that special commissioners should have power to confirm those guilds in the possession and enjoyment of their property which devoted it to useful and charitable purposes, and that it was only sought to suppress institutions cultivating religious superstitions and corrupt practices, very few of the guilds were spared.

1500—1700. In the 16th and 17th centuries some of these institutions which had survived the preceding period, and still retained sufficient cohesion, were reformed, whilst many new societies were started, embodying some of the features of the guilds, but in conformity with more modern ideas. As the organization of the trade guilds, following the general tendency of the times, gradually changed, the various trade societies, having vastly increased in numbers and become more democratic, re-united themselves into aid societies, which combined in themselves some of the characteristics both of the religious and the trade guilds. The relief and economic system of these societies, moreover, underwent a radical change, in the direction of requiring fixed contributions to be made by the members for the grant of specific benefits. As an illustration of the financial basis of such societies, it may be interesting to note that in one society the subscriptions were 2*d.* weekly, to provide for benefits, during sickness, of 6*s.* weekly for the first year, and 3*s.* 6*d.* per week for the remainder of illness, while a levy of 6*d.* per member was made at each death.

About the beginning of the 18th century a movement arose in the Guild of Masons which exercised a great influence in stimulating the rapid growth of friendly societies. By the introduction of mystic rites and observances, and by other changes which need not be detailed here, the Guild of the craft of Masonry was transformed into the order of Freemasons. Members of the aristocracy joined the new and highly successful order, while the mystery with which they surrounded their transactions took the public fancy, and created desire in other societies to imitate them. The Odd Fellows are known to have existed shortly after this period, though in a form having little or nothing in common with their modern development, the oldest lodges being practically social clubs.

Number and class of Societies.

The friendly societies existing at the close of the eighteenth century appear to have been very similar in the nature of their operations to those of the present day. It is difficult to estimate, with any degree of certainty, the number of societies or their aggregate membership, but there is good reason for believing that they were at least as numerous in proportion to population as at the present day. Sir P. Colquhoun, in 1796, estimated the number of societies in London at 600, with a membership of 70,000, while Sir F. M. Eden, five years later, considered that the whole kingdom might number 7,200 societies, with a membership of over 600,000. The amount of the sick pay allowed seems somewhat larger in proportion to wages than now, 6*s.*, 8*s.*, and 10*s.* a week being common sums; while the modern system, now all but universal, of reducing the sickness allowance in cases of prolonged illness, appears to have been at this date rare, although, in some cases, the allowance ceased after a specified period of 13, 20, or 26 weeks of illness, and in others provision was made for the superannuation of members at a reduced rate of pay.

The influence of occupation upon health—about which so much has since been written, but which is still largely an unsolved problem—was not unrecognized. In Gloucester, we find societies who excluded workers in white lead, miners, and others; and similar precautions were not uncommon. Societies confining their membership to distinct trades or classes were relatively more numerous than at the present day, and naturally so when it is considered to what an extent the present friendly society system has grown from such trade societies. The propriety of limiting the age at entry was very generally recognized, and endeavours were often made to secure the permanency of societies by making the amount of the benefit depend upon the funds in hand. The provision for death benefits by means of levies upon the members at each death—no doubt a method of great antiquity—was also common, while in many societies pensions to the widows of members took the place of the now customary funeral allowances. It was very usual for societies at this period to limit their membership—101, 81, and in some cases 51 or 41 members, was made the limit of a society: though what superstition lay in the use of these numbers it is impossible to say.

The rules of the societies, as in earlier times, almost always

exercised a paternal care over the morals and even the opinions of the members.

Defects of System.

The friendly society system on the whole worked well, but it had many defects which became increasingly apparent. The societies having no legal status, were open to fraud and corrupt practices, no prosecution being possible except in the names of all the members, the societies being regarded at law as partnerships. The uniform premiums did not answer well; as members grew old, younger men did not care to join, and bankruptcy often resulted. There was also too much feasting by members at the expense of the funds.

Nevertheless, while the State poor laws, which, in 1572, had replaced the administration of charitable funds by the Church, had proved to a large extent a failure, voluntary efforts had succeeded, in spite of disadvantages, in nursing and gradually extending over the country that system of friendly societies, to a larger extension of which many looked for a removal of the evils created by the poor laws. While some reformers favoured the continuance of the voluntary system under the control of the parishes, others advocated compulsory and universal State insurance. Defoe, in his essay on *Projects*, had, as early as 1697, recommended the establishment of pension offices in each parish. Francis Maseres, in 1772, formulated a scheme for establishing life annuities in parishes for the benefit of the industrious poor, an idea which, in the following year, was embodied in a Bill passed by the Commons but thrown out by the Lords; while about the same time various projects for compulsory State insurance were also before the public.

In 1787 Gilbert originated the idea of requiring each society to be registered in order to obtain legal protection; and it was his proposals undoubtedly which induced the framing and passing of the highly important Act of 1793, which practically settled the struggle between compulsion and voluntaryism in favour of the latter system.

Act of 1793.

The Act of 1793 marked a new era in the history of friendly societies. Their legal position before the Act, and the various abuses that resulted from it, have already been referred to. The Act remedied these abuses by establishing some supervision and some safeguards as to the management. It required, for example, the submission of rules, and alterations of rules, to the justices at Quarter Sessions, who had power to confirm them or not at discretion. The objects of the society were to be

specified, and a dissolution could only take place with the consent of the justices. On the other hand, certain privileges were conferred upon the societies. They might appoint officers; trustees or treasurers might give a bond to the clerk of the peace; free application was allowed to the Court of Chancery to compel trustees to account for moneys, and a first claim was given on their estates. The power was also conferred to allow of disputes being settled by arbitration. The Act was extended in 1795 for the benefit of societies existing prior to 1793, and in 1796 an Act embodying similar provisions was passed by the Irish Parliament. The extent to which the Act was utilized differed very much in various parts of the country. On the whole, probably more than 50 per-cent of the societies availed themselves of its provisions. Very many, however, were much frightened by the new legislation, and in a large number of cases societies were dissolved.

It is not easy to systematize the various efforts made by the Legislature to control or to encourage, as the case might be, the friendly society movement between 1793 and the great Act of 1875. All of these attempts were necessarily in the nature of experiments, and were generally without any common plan.

The Acts of 1803 and 1809 can scarcely be said to have been important. In 1818, however, a Bill was introduced providing for the appointment of five public valuers—in the language of the Bill, "persons skilled in arithmetical calculations"—to whom tables and rules might be submitted at fixed fees. The fear of Government interference, however, which was so common among members of friendly societies, caused this provision to be struck out of the Act of the following year (1819). This Act provided, however, that rules and tables of all future societies should both be confirmed by the justices at Quarter Sessions; and, moreover, contained the important provision that such justices should not confirm or allow any table of payments or benefits, or any rules dependent upon or connected with the calculations thereof, until it shall have been made to appear to such justices that the said tables and rules are such as have been approved of by two persons at least, known to be professional actuaries or persons skilled in calculation, as fit and proper according to the most correct calculation of which the nature of the case will admit. In the same way, dissolution could only take place with the consent of the trustees and under the certificate of two actuaries. Several provisions were made as to the investments of societies, permitting funds to be placed in savings banks, in the

Act of 1819.

Bank of England, on debentures, in the public funds or real security. Investments in savings banks, to the extent of £300, were permitted nine years later, by the Act of 1828.

Act of 1829. Previous Acts repealed.

The Act of 1829 contained several provisions of considerable importance. By this Act, nearly all previous legislation relating to friendly societies was repealed, and the law consolidated. The Act allowed any number of persons to form a society and to make rules, provided that these latter should declare the purpose of the society, and that they should be submitted to a barrister or to the Lord Advocate for certification. When so certified, they were to be confirmed by the justices at sessions and to be deposited with the clerk of the peace. By the 6th section of the Act it was provided that the justices should not certify the rules unless they were satisfied with the tables proposed to be employed. The rules, when entered and deposited, were binding on the members, and could only be altered at a general meeting of the society, the place of such meeting being specified in the rules. Societies were allowed to appoint committees of management, whose powers were to be declared in the rules, and who were controllable by the members. Treasurers or trustees were to lay out any surplus contributions, to render accounts, and pay over balances, while the priority of claim upon the estates of such trustees was continued. Provision was also made for the payment of sums not exceeding £20, where members died intestate, to persons appearing to be next of kin. Societies were to make annual audits and statements of the funds to the members; and a dissolution could only take place when agreed upon by five-sixths of the members and all persons entitled to relief. Minors were allowed to become members, and to act as such.

Quinquennial returns.

Perhaps the most important provision in the Act however, was that requiring quinquennial returns of sickness and mortality to be made to the clerk of the peace, by whom they were to be transmitted to the Secretary of State, and laid before Parliament; all societies neglecting to make such returns losing the privileges of the Act. The Act embraced all existing and future societies, the former, if enrolled, being required to conform within three years (a period, however, which was subsequently extended) to the provision for quinquennial returns of sickness and mortality. The object was stated clearly in the Act to be the collection of data by which tables of the payments and allowances dependent on the duration of sickness

and probability of human life may be constructed, "the present existing data of these subjects having been found imperfect and insufficient." The form of return was to state, among other things, each member's occupation, date of birth, of admission into the society, and of becoming free to benefit, with the number of weeks' pay received in each of the five years on account of sickness and of superannuation. A new feature of the Act was the absence of any limit as to the sum which a society might insure on the death of a member, one effect of which was the enrolment of various societies which were, strictly speaking, insurance societies —as the Clergy Mutual—which added the business of sickness assurance to that of an ordinary mutual life office.

Act of 1834. By the Act of 1834, the powers of the justices to decline to certify the rules, in cases in which they might deem the tables insufficient, was withdrawn, as also was the priority of claim by societies upon the effects of trustees and others. The purposes for which societies might be formed, however, were somewhat extended, and were stated to be "for the mutual relief "and maintenance of all and every the members thereof, their "wives, children, relations, or nominees, in sickness, infancy, "advanced age, widowhood, or at any other natural state or "contingency whereof the occurrence is susceptible of calculation "by average, or for any other purpose which is not illegal." It was carefully provided, however, that for any such general purposes, not being "natural contingencies", the contributions should be kept distinct and separate, or that the charges for these benefits should be defrayed by levies upon the members.

Act of 1846. The quinquennial returns were continued. The Act of 1846 imposed a penalty for failure to forward these returns, and appointed a salaried registrar, who was only to register rules when tables were certified by an actuary.

The Act of 1850 required this certificate in respect of annuity tables only, and limited the amount of benefits, especially in the case of children's assurances, allowed investments with National Debt Commissioners at a fixed rate of interest of 2*d.* per-cent per diem, and allowed branches to register. This year was memorable on account of the founding of the well-known Royal Liver Friendly Society, which within 20 years reached a membership of over half-a-million.

Most of the features of the recent legislation were continued by the Acts of 1855, 1858, &c., the position of friendly societies remaining very much the same down to the important legislative

enactment of 1875. This was preceded by the Royal Commission of 1870, the result of whose labours is embodied in four successive reports of the utmost value to every student of the friendly society system.

Act of 1875. It is only possible to give a brief outline of the changes effected by the great Act of 1875, by which societies are now governed. By this Act the law relating to friendly societies was both consolidated and amended.

The following provisions of the new Act may be mentioned as among the most important:

The maintenance of the voluntary principle, as respects registration and the actuarial certification of tables.

The continuance of the quinquennial and annual returns.

The limitation of benefits, especially for infants' assurances, this provision also applying to industrial assurance companies.

The authorizing of the Treasury to issue regulations from time to time, and the power conferred on the registrars to appoint inspectors, when duly requested to enquire into the management of societies, and call general meetings of the members.

The compulsory valuation of all registered societies every five years, and the provision for the appointment of public valuers by the Treasury, who might undertake such valuations.

The reconstruction of the Registry Office, resulting in the appointment of an actuary to the Registry.

The Act on the whole was drawn with great care, and upon undoubtedly sound principles. A few amending Acts of no great importance, have been passed since. The third of these dispenses with the quinquennial returns of sickness and mortality which had gradually accumulated a mass of data more than ample for the purpose for which they were collected. These materials have for some time been undergoing complete examination and analysis at the able hands of the actuary to the Registry. The results of the analysis, from the improved form in which the returns have been made, and the care that has been taken to eliminate doubtful returns, may be expected to throw much light upon the rates of sickness prevailing among the

friendly societies of this country, and at the same time solve some of the important questions (such as the duration of sickness, for example) which have hitherto been left in some obscurity.

Existing Societies.

Classification.

The friendly societies of the present time are not more remarkable on account of their number than in respect of the great variety to be found in their constitution and mode of working, and minute as is the classification adopted by the Commission of 1875, it is very doubtful if it is absolutely comprehensive. Such a classification of societies may be viewed from two stand-points—either as regards their method of management and the scope of their operations, or in respect of the benefits granted and the method in which those benefits are provided for. The classification of the Commission of 1870 proceeds mainly upon the former basis. In what follows I have endeavoured to combine both these points of view. As may be supposed, however, there is considerable difficulty in drawing a hard and fast line between some of the classes enumerated.

1. *Local Friendly Societies* (in town or country), independent as to management, and usually granting benefits in cases of sickness and death (occasionally with medical attendance), form the most ancient, as well as one of the most interesting, of existing classes of societies. They abound in every town, and in almost every considerable village, throughout the country, and, notwithstanding the recent rapid growth of the affiliated orders, constitute a very large proportion of the registered societies.

They may be subdivided into:

(*a*) Societies having uniform contributions and benefits for all ages of entry, the limits of which vary considerably. An entrance fee, increasing with the age at entry, is the usual and quite inadequate substitute for a graduated scale of contributions. There is usually no special restriction as to the occupations from which the members may be drawn, which are, in general, those prevailing in the district. Where these are mainly of one class, the society will approximate to the particular trade societies referred to under (*d*).

The vast bulk of these local societies are in a more or less unsound financial position, the contributions

being generally not more than adequate at the lowest ages at entry. Some cases, however, might be cited, in which such societies have been extremely well conducted and have been very successful. A remarkably interesting group of such cases occurs in the small societies principally in East London, which appear to have been founded by the original French Protestant refugees, about 1685 and subsequently.

(*b*) Societies similar to the last named, in which the funeral benefits are met wholly or in part by levies.

(*c*) Societies having a graduated scale of contribution, in most cases adopted under actuarial advice, or in imitation of other societies that have taken that step.

(*d*) Societies confined to particular trades (whether rigidly so by virtue of the society's constitution, or practically so in consequence of the locality and prevalent occupation of the inhabitants, is of little moment from an actuarial point of view). These societies are extremely numerous, but the nature of their benefits and general working differ but little from that of the ordinary local society. They are of interest to the actuary as affording examples of societies to which the ordinary table of sickness and mortality cannot be generally applied.

2. *Large General Societies,* whose membership extends over a large area, although the management is usually in one location. These societies may or may not have agents, whose duty is to obtain new members and collect subscriptions.

As typical societies of this class may be mentioned the London Friendly Institution (established in 1824), the Royal Standard (four years later), the Royal Oak Benefit Society (in 1837), the United Kingdom, and the Hearts of Oak. Some of these societies have agents; others, as the Royal Standard and the Hearts of Oak, dispense with this aid outside London, the members remitting their contributions and receiving benefits by Post Office Orders. This latter system tends, naturally, to economy of management, and in the latter of these societies the expenses amount to only about 6 per-cent of the total expenditure.

Societies of this class have, almost invariably, extensive graduated tables of contributions, and benefits are generally guided by actuarial advice, and, on the whole, their financial position is satisfactory.

3. *Patronized County* and *District Societies* form a special class as regards management, which rests wholly, or in great part, in the hands of the honorary members. The societies are usually spread over a considerable area, mainly in rural districts, and although their experience has varied considerably, they have, on the whole, been well and successfully managed. It has been said that their constitution does not enlist so strong a feeling in their members against imposition, and hence it is often difficult to keep down the sickness claims to within a reasonable amount. Whilst this may be to some extent the case, much of the excessive sickness, especially the heavy claims for reduced allowances at the older ages, is doubtless to be attributed to the class of occupations generally followed by the members, which is largely agricultural.

4. *The Affiliated Orders* represent the most popular and growing section of friendly societies. The Manchester Unity and the Ancient Order of Foresters each number over half-a-million of members, spread throughout the whole of the country, and their numbers are constantly and rapidly increasing. The various branches are generally independent in respect of sick pay, but mutually dependent in the limits of each district as to the liability for funeral allowances; while the districts again are independent of each other financially, but unite for legislative purposes. In addition to the sickness funds belonging to each lodge or court, there is usually a district burial fund, which is kept up by equal levies for each member in the district, irrespective of age; and, in some orders, general funds common to the whole order, as the general funeral fund and the annuity fund of the Nottingham Odd Fellows, and the funeral fund of the Derby Midland Odd Fellows. In many districts, again, widow and orphan funds provide a fixed sum to the widow, and a smaller sum on account of each child of a deceased member. These funds are, in many instances, it is to be feared, insolvent. There is, of course, no mutual responsibility between the several branches of the affiliated orders. The average size of the lodges is small, averaging, in the Manchester Unity and the Ancient Order of Foresters, a little over 100 per lodge.

The extent of the various orders in this country will be seen, from the following approximate statement, to be very great, and

there is no doubt that they are at the present time spreading rapidly:

TABLE 1.—*Affiliated Orders in the United Kingdom* (1886).

Order	Members	Funds
		£
Ancient Order of Foresters . .	582,000	3,342,000
Manchester Unity of Odd Fellows .	542,000	5,053,000
Grand United Order of Odd Fellows .	83,000	273,000
Six Orders of over 40,000 members .	316,000	1,100,000
Ten Orders of over 10,000 members .	160,000	608,000
Smaller Orders (estimated) . .	70,000	170,000
Juvenile Societies (estimated) . .	150,000	150,000
TOTALS	1,903,000	£10,696,000

5. *Dividing Societies,* an important class of society as to numbers, and especially numerous in Scotland, but of little interest to the actuary. The plan of these societies is to receive certain fixed annual contributions from the members, out of which all sickness claims for the year are paid, together with the expenses of management, while any surplus that may exist at the end of the year is returned to the contributing members, in some cases wholly, in others after a deduction has been made for the purpose of starting the new year's transactions. Since all the members pay an equal contribution, irrespective of age, great difficulties must attend the progress of these societies, and it is only in very exceptional cases that they can last for many years. Their chief difficulty is with prolonged sickness, and it is very usual to avoid the payment of superannuation by advancing either the whole or part of the anticipated death benefit and excluding the member. It is worthy of note that the principle on which these societies are conducted is very similar to the American mutual assessment scheme of insurance. The stoppage of these societies is usually brought about by the failure to obtain the constant influx of new members at the younger ages, upon which they must depend to keep down the average sickness rate.

6. *Deposit Societies,* which, like dividing societies, can scarcely come within the cognizance of the actuary, being rather savings' banks than friendly societies. In the language of Mr. Best, who may be said to be the originator of this class of society: "A member's money is and remains his own under his own

"control, under such limitations only as are necessary for the "protection of all."

7. The large *Collecting Burial Societies* form a very important class of societies, differing totally in character from any of the preceding classes. In the aggregate, they probably include nearly a million and a half members, an extent of membership nearly equal to that of the affiliated orders. The largest societies are general in their character, their business extending over the whole of the kingdom. Some few include sickness benefits in their operations, but this part of the business has never been successful, from the extreme difficulty of working sickness business over a large area. Nearly all the largest societies are located in the big towns, such as Liverpool, Manchester, and Glasgow. The largest society of all, the Royal Liver, has a membership equal to that of the Manchester Unity, though of course with much smaller funds. The whole business of these societies usually consists of insuring small sums by weekly payments, an immense proportion of the insurances being on the lives of infants. The premiums are collected weekly by the agents of the society, which necessarily makes the ratio of expenditure very heavy. In this respect they approximate very closely to the industrial assurance companies; indeed, in the class of business undertaken, and in the general management of the societies, there is scarcely any distinction to be drawn. Of their financial position, on the whole, it is difficult to speak with certainty, though the larger societies are probably solvent.

8. *Annuity Societies* form a class entirely distinct from the ordinary type of friendly society. They are generally of old standing, very few now being registered, perhaps on account of the requirement of an actuarial certificate. They, however, exist in some numbers in the West of England. The funds of these societies are naturally very large in proportion to their membership.

9. *Subsidized Societies,* among which the large railway funds form an important element. These are, of course, of comparatively modern growth, but they have now become very general. In most cases membership is compulsory, and, in addition to the contributions made by members, they are frequently supported by a subsidy of a fixed amount per annum from the company; notwithstanding this extraneous assistance, however, many of the funds are in an unsatisfactory financial position. In addition to providing for sickness benefits, an important feature of these

societies is the provision for retiring allowances. The experience of the societies, in respect of both mortality and sickness, is often very different from that of ordinary friendly societies.

Most of the railway societies have dropped out of registry owing to the companies having taken powers under their special Acts for regulating the provident funds attached to them. The system of compulsory membership, which is common to these societies, lends a certain stability to their operations, as the distribution by age of the members probably varies very little from year to year. The same feature is found in many of the societies attached to collieries, iron works, and large factories. These societies, existing amongst widely different classes of occupation, have special interest for the actuary, and the experience derived from their working is calculated to throw considerable light on the laws of sickness and mortality as affected by occupation; many being limited to callings of more than ordinary risk, such as those of fishermen, pilots, and so on, in addition to those of miners and railway servants already mentioned.

10. *Societies of Females* also form a special class by themselves. In some cases females are admitted as members in ordinary friendly societies, but it is doubtful whether this plan has worked satisfactorily, as, in many cases, the rates of sickness among the female portion of the society have been heavier than among the remaining members. There are several orders composed exclusively of females, and constituted on the same principle as the larger affiliated orders. The benefits frequently differ from those in male societies, and usually include allowances on confinements.

Number and Financial Position of Registered Societies.

It is difficult to estimate exactly the numbers of even the registered societies, but they may probably be put down at something like 25,000, of which more than one-half are branches of the various affiliated orders. The aggregate membership of these societies, when the large burial societies are included, probably exceeds 4,000,000. The materials do not as yet exist for forming a just opinion as to the financial position of the societies as a whole. In the paper read before the Statistical Society, 21 April 1885, Mr. E. W. Brabrook, Assistant Registrar of Friendly Societies, gives a table showing approximately the results of the valuations recorded during the five years ending 31 December 1880. According to this estimate, the aggregate valuation balance sheet of the 6,567 societies valued would be as follows:

TABLE 2.—*Aggregate Balance Sheet of* 6,567 *Societies.*

LIABILITIES.	£	ASSETS.	£
Present value of benefits .	33,405,152	Amount of funds . .	8,380,851
Other liabilities . .	3,030,582	Present value of contributions	23,066,592
Surplus (in 1,537 societies)	885,382	Other assets . . .	717,857
		Deficiency (in 5,030 societies) . . .	5,155,816
	£37,321,116		£37,321,116

It is not possible to draw any very decided conclusions from these figures, from the uncertainty attaching to the large number of valuations made by unskilled valuers. They probably roughly indicate, however, the financial position of the societies as a whole, as it would be brought out by a general valuation upon the basis of the Manchester Unity experience, at about $3\frac{1}{4}$ per-cent interest. While the proportion of the net deficiency to funds is undoubtedly large, it is the opinion of those best qualified to judge, that the financial position of the societies generally is annually improving, not so much, perhaps, by the progress of individual societies, as by the gradual dying out of the unfit and their replacement by their sounder and better-managed rivals.

The Colonies. During recent years friendly societies have taken considerable hold in the various colonies, and especially in Australia. Branches of the various affiliated orders constitute the great bulk of these societies, the Manchester Unity of Odd Fellows having been in existence in the Australian colony since 1840.

The duties which in this country are vested in the Registry Office are, in Victoria, divided between the registrars and the Government statist. In the Report of 1884, some very interesting particulars are given as to the valuations received up to the end of that year. In all, 660 societies, comprising 43,000 members, had made valuations: less than one-fourth showed a surplus, and the remainder a deficiency—the net deficiency for the whole being about £300,000. As compared with the Manchester Unity experience, it is stated that the actual mortality in the societies had been less than the expectation by 9 per-cent, but the sickness was greater by 2 per-cent, while the withdrawals had been almost three times more numerous. Thirty-one parent societies were in existence at the close of 1884, comprising 788 branches and 59,000 members, showing an increase of 4,000 on the previous

year. The income of the societies amounted to £228,000, the expenditure to £183,000, and the total funds to about £570,000, invested at an average rate of $5\frac{1}{8}$th per-cent. This high rate of interest must have an important bearing on the financial position of the societies, although there seems little probability that it will be permanently maintained.

In New Zealand, the affiliated orders also appear to be fairly numerous, and to constitute nearly the entire bulk of the societies there existing, the Manchester Unity and the Ancient Order of Foresters taking the lead. At the close of 1883 there were 243 societies, including lodges, with a membership of 19,000, and funds of £230,000, an average of a little over £12 per member. The societies are small, averaging between 70 and 80 members. Mr. Edmund Mason, the registrar, furnishes, in his report, some very interesting particulars as to the rate of mortality and sickness prevailing. The following is an extract from his table relating to the experience of the New Zealand societies for the ten years, 1873 to 1882:

TABLE 3.—*Experience of New Zealand Friendly Societies.*

Age of Members	Percentage of Members Sick per Annum		Weeks' Sickness per Annum per Member		Deaths per 1,000 per Annum	
	New Zealand, 1873-82	A.O.F. 1871-75	New Zealand, 1873-82	A.O.F. 1871-75	New Zealand, 1873-82	A.O.F. 1871-75
20 to 25	12·69	21·5	0·55	0·82	4·18	7·43
25 „ 30	11·72	21·6	0·60	0·85	4·71	7·29
30 „ 35	13·38	22·3	0·73	0·97	5·28	8·86
35 „ 40	14·77	23·4	1·01	1·15	7·04	10·92
40 „ 45	15·21	25·0	1·17	1·37	10·70	12·84
45 „ 50	15·40	26·9	1·33	1·71	9·90	16·58

He rightly recognizes, however, that the statistics of the New Zealand societies are not yet sufficient as a basis of calculation for contributions. It will be seen that, as is the case with the Victoria societies, the sickness rates are, throughout, higher than in the English tables, while the mortality is considerably lower. The financial effect of this feature, however, will be much more than counteracted, for many years to come, by the high rate of interest at which the funds are invested, which is stated to amount to over 6 per-cent. By the Act of 1878, annual returns of sickness and mortality are required, the cost of which is defrayed by the Government.

Generally societies in the colony are governed by the Act of 1877, and the Governor's Regulations issued under it, which closely approximate to our own Acts of 1875 and 1876, and to the Treasury Regulations, except that some of the duties performed by the English Registry Office are, in New Zealand, divided between the registrar and the revising barrister. One very important provision of the Act secures that every valuer shall be approved by the Governor, and another provides for the absolute separation of the management and benefit funds.

In New South Wales, Queensland, and South Australia, the position of the societies is very similar to that in Victoria and New Zealand.

Friendly Societies in Foreign Countries.

France.

In no part of the Continent has the friendly society system taken root so extensively as it has in this country, and, so far as can be ascertained, the system is not only less extensive, but, on the whole, less ancient. Government patronage and control is carried out to a much greater extent than here, and much of the legislation is of a character that would be decidedly resented by the societies in this country.

In France the societies seem to have taken their origin mainly from religious associations or guilds rather than from trade societies, as here. They appear to have existed extensively amongst the French Protestants, as the refugees who came to this country after 1685 introduced several, some of which have been in existence down to quite recent times; but the number of societies remained very limited until the middle of the present century. In 1849 a committee was founded for the encouragement of provident societies, which did very useful work by making extensive inquiries into the then condition of societies in France. One result from the labours of this committee was the publication, by M. Hubbard, the secretary of the committee, of a report, dealing not only with the condition of the societies as a whole, but also containing an investigation into the sickness and mortality experienced by 25 societies who responded to the invitation to send in returns. The results of this investigation, which have been made known to actuaries in this country through the medium of Mr. Samuel Brown's paper (*J.I.A.*, v, 208), are of considerable interest. In many respects they are similar to the experience obtained by the Highland Society 25

years previously, the sickness being lighter than that shown by any experience in this country. The experience was divided into sickness claims and claims for infirmity, a distinction answering nearly to bed-lying and walking pay in some of our own societies, or perhaps better to the distinction made between sickness and superannuation—the reduction in the allowance for the latter being very considerable.

After the establishment of a superior commission for the encouragement and supervision of friendly societies in 1852, the number of societies in France rapidly increased. By the decree of that year they were divided into two classes—approved societies and allowed societies. The difference between the two classes of society answers very nearly to that existing between registered and unregistered societies in this country. Mr. Ludlow gives the following list of privileges conferred by "approval":

1. The right of "civil personality" (practically equivalent to our "incorporation"), carrying with it the right of taking immovable property on lease, of accepting gifts and legacies (with the prefect's authority) up to £200, and by decree of the Council of State for higher sums.
2. The occupation, rent free, of places of meeting, put at their disposal by the communes.
3. Supply of the necessary books for their management and accounts.
4. A reduction to one-third of funeral taxes.
5. Exemption (generally) from stamp duties.
6. Deposit of funds, without limit, with the "Caisse des Dépôts et consignations" at 4½ per-cent interest, the grant of life-annuities from the same office, and the right of depositing up to 8,000 francs (£320) in savings banks, postal or other.
7. Contributions from the funds of the "Caisse des Dépôts et consignations" in proportion to payments in collective insurance in case of death.
8. Admission, at reduced rates, for convalescents in the Department of the Seine in two municipal establishments.

Honorary members form a special feature of the approved societies. The management is mainly vested in them, an important

post in these societies being that of president, who, in all cases, is appointed by the Government, the wishes of the members of the society being, of course, considered. In carrying out the rules, however, all propositions are submitted to discussion and to the votes of general meetings.

The difficulty of providing for sick claims and superannuation at the older ages, which has been so seriously felt in societies in our own country, was also in existence among the French societies. Among the old societies the pension fund, generally, rested on an unsound basis, offering benefits to the members which there was no prospect of their ever receiving. An important change in this respect was made by the decree of 1852 already referred to, which substituted for the old system of pension funds the power of taking a certain portion of the reserve fund, especially the contributions of honorary members, with a view of assisting the aged and infirm. In the case of approved societies, this portion of their funds is placed at the *caisse* aforesaid. Accounts are opened in the name of each society which pays in, and the interest of the moneys is distributed in the shape of annuities among the aged members, who are chosen by a general meeting, and who have been at least 10 years in the society. At the death of these members the capital of the pensions is returned to the society and used again in the same manner for the benefit of the other members.

The average magnitude of the societies appears to be quite equal to that of the English; reduction in the rate of pay after six, nine, or twelve months, is now almost invariable, and, as already intimated, is very considerable; and in addition to members who are permanently invalided, there are regular pensioners whose benefits date from a certain fixed age, and who receive similarly reduced amounts.

Since the period at which friendly societies in France first became directly encouraged and patronized by Government, their numbers have rapidly increased. In 1884, the total number of societies was 7,743, of which 5,570 were approved. Of these latter, 3,956 consisted solely of men, 1,452 of men and women, and 162 of women only. The number of members in approved societies was stated to be 928,788, and in authorized or allowed societies 319,123. The funds of the approved societies amounted to about £3,860,000, and of allowed societies to £1,050,000, in addition to investments for annuities belonging to approved societies only, and amounting to some £2,170,000,

making a total for all societies of something over £7,000,000 sterling. This, although a large sum, is much below the amount standing to the credit of English societies.

Friendly societies in France still appear to be without any trustworthy guide in the way of sickness and mortality tables deduced from their own general experience; but there is no doubt that their financial position, as in our own country, has greatly improved during the latter part of the present century.

Belgium. There is in Belgium only one legally recognized class of friendly societies, answering to the approved societies in France, but the unrecognized societies exist in large numbers, and many of these, unlike the unregistered societies in our own country, furnish returns to the Commission, which takes the place of our Registry Office. From the report of M. Duepetiaux, a member of the Central Statistical Commission, in 1852, it would seem that the origin of many of the Belgian societies is very ancient, and their organization of great variety. It appears that there were then only 211 "societies of mutual aid" existing in Belgium, numbering 24,300 members.

Societies are regulated by the law of 3 April 1851, which authorizes their recognition by Government; such recognized societies enjoying the following advantages:—The power to sue or be sued; exemption from stamp duties and registration fees of all deeds; power to receive donations or legacies of personal property; and other less important privileges.

The number of societies in Belgium does not appear to have been increased materially during the 30 years that have elapsed since the law of 1851. At the close of 1882, according to the report of the Permanent Commission of Friendly Societies, there were 191 recognized societies, with about 30,000 benefit and 4,000 honorary members, while there were 77 non-recognized societies which had forwarded reports, with a membership amounting to 23,000. It is stated, however, that in addition to these, the mining population had special societies, with an aggregate of over 104,000 members.

The average contributions in the recognized societies amount to about 9*s.* per member, and in the non-recognized about 12*s.*; and the average sickness benefit to about 1*s.* 2*d.* in the former, and 1*s.* in the latter class per day.

The practice of valuations appears to be unknown, and no attempt has been made to investigate the experience of the

societies as to sickness or mortality.* It is stated, however, as a result of experience, that the monthly contribution should be equivalent, or nearly so, to the daily pay in sickness; and it appears that the Commission does not recommend for recognition societies whose contributions it considers insufficient. Dividing societies are not recognized.

In addition to the friendly societies, properly so-called, there exist provident societies for the assistance of sick or disabled workmen, their wives and children, whose operations are much more varied than those of the friendly societies. Thus, provident funds have been founded for miners, servants on the State railways, fishermen, pilots and sailors. Of these the most important are, perhaps, the miners' funds, which provide pensions either for life or for shorter periods. The life pensions are allotted to workmen who may be permanently disabled, to the widows, and, in some cases, the parents or grandparents of workmen who have perished by accidents in the mine—the temporary pensions being granted to the infant children, or orphans of such miners, and ceasing when the children attain the age of 12 years. The funds are derived from the following sources:—A deduction from the wages of the miners, varying from a half to one per-cent; a similar contribution from the proprietors of the mines; allowances granted by the Government or the province, in addition to interest on capital gifts and legacies. The assistance on the part of the proprietors is voluntary.

So far back as 1838, a fund was established for workmen employed on the State railways, which is supported by a deduction of about 2 per-cent from the wages of the men, and which grants assistance to sick members varying from one-half to three-quarters of their wages, or to the whole wages in case of wounds received in the service, a graduated scale of benefits being in force, dependent upon the extent of the family of the injured man.

Germany. The objects of the German friendly societies are, by the Act of April 1876, confined to relief in sickness or convalescence, medical aid, and allowances to the families of deceased members which take the place of the death benefits usual in our own societies. It is provided that the rules shall contain no provision not connected with the objects of the society, and there are provisions for quinquennial valuations, returns of sickness and

* The report for 1859, however, gives the rate of sickness among the members, from 18 to 50 years of age, as 6·81 days, a result stated to be midway between the French and English experience.

mortality, and balance sheets, very similar to our own. The leading differences are that the amount of Government interference is considerably greater, and many points, in this country left to the discretion of societies, are, in Germany, fixed by law. For example, the period after which a member becomes free to benefit must not exceed 13 weeks, while the right to relief exists for a similar period after withdrawal or exclusion. The same period is fixed as the minimum duration of relief, should sickness continue so long. The limits between which the sickness benefit may vary are also fixed, and depend upon the wages of the members, while societies are not allowed to refuse the benefit for particular cases of sickness.

All societies, whether recognized or not, are considered to be under superintendence, and the authority appointed for this purpose by the provincial government may inspect the books of the societies, receive yearly statements, and give notices to societies (entailing dissolution when disregarded) with respect to arrears of contributions or benefits, illegal resolutions, &c., may appoint liquidators, call general meetings when the committee of management fails to do so, and impose fines for failure to forward returns.

The higher administrative authority registers rules and receives quinquennial returns of sickness and mortality, and valuations, &c., and, in certain cases, subject to appeal, may order the closing of the society. It is, moreover, the superintending authority for all federations of societies, whose boards of direction sit within its jurisdiction.

In the chief registrar's report for 1876 he makes the following remarks upon the German legislation of that year:—"It is not "too much to say that, under such a weight of official oversight, "any multiform development of our friendly societies would have "been impossible. Yet these are the provisions relating to what "may be termed the free societies. The fact must now be "recalled that these form only one portion of the German system. "According to the law of the 8th April, amending the 'Gewerbe "Ordnung', the formation of friendly societies may be ordered "by local regulations. In this case, the communal authorities, "after hearing the parties interested, appear to frame the rules "of the society, and by local regulation it may be made com- "pulsory on all working men aged 16 years and upwards to "join it, unless already members of some registered society, and "in like manner compulsory on all employers to advance the

" contributions of their workmen to the society so established by " the commune, up to the amount of one half of their wages, and " to subscribe themselves to the society, up to the amount of one " half of the contributions of their workmen. The same powers as " those given to the communes are given also to 'larger communal " unions.'"

In 1882 a law was passed rendering insurance against sickness obligatory on all persons whose wages do not exceed 4*s*. a day, employed in mines, factories, and some other undertakings, and upon agricultural labourers when so determined by the district authorities, two-thirds of the contributions being paid by the workmen, and one-third by the employers, but the latter being responsible for the whole.

From a Minute of the German Government which has just appeared (November 1887) in the official papers, it seems to be the intention to extend the existing legislation so as to cover superannuation allowances for all persons of 70 or over, and for all below that age who may become permanently unable to work.

Switzerland. Societies in Switzerland are very numerous in proportion to the population, and appear to be somewhat similar to the German societies, the majority providing only for sickness allowances, and not funeral benefits. The average magnitude of the societies is very similar to that obtaining in this country, ranging from about 100 to 120 members. In 1883 a proposal for compulsory insurance of the whole population, for both sickness and burial, was made in the Great Council of Basle by the Commission which had been appointed to consider the subject, but was rejected. The Executive Council, however, was directed to prepare measures for gratuitous burial and compulsory sickness insurance in the case of the adult labouring population.

Denmark. The guild system common to nearly all European societies, was, in Denmark, succeeded by one of compulsory friendly societies, corresponding to that of the German Zwangskassen. After the enactment of the Free Constitution of 1849, however, it was considered unnecessary to maintain a compulsory principle, and free friendly societies were allowed to be established with the funds belonging to the compulsory ones. Very few restrictions being imposed by the law on these societies, a large number were founded, and, at the present time, out of a population of 2,000,000, probably about 150,000 are members of these societies. Affiliated orders like the English, do not

exist. The most recent statistics with respect to Danish societies appear to be those contained in the publications of the Statistical Bureau for 1885, and which give particulars of the operations of the societies to the close of 1882. There were at that time 831 societies, with 115,000 ordinary members, and between 5,000 and 6,000 honorary members. The income of the societies amounted, in the aggregate, to some £58,000, against an expenditure of £53,000, and the funds in hand to a total of £118,000 (about two years' income), and during the five years had risen from about 17*s.* 9*d.* per member to £1. 0*s.* 5*d.*

Attempts have recently been made to investigate the sickness rate prevailing among the Danish societies, but it is stated that the materials are scanty, and, it is feared, incorrect, while the experience of different societies is extremely dissimilar. The following table, however, may be found interesting, as giving a comparison of the results obtained with the rates prevailing in the Manchester Unity Society:

TABLE 4.—*Sickness Rate in Danish Societies.*

Age	Danish Societies (1878-82)		Manchester Unity (R. T. & C.), 1866-70
	Rural and Town	Copenhagen	
20–29	·48	·75	·78
30–39	·54	·94	·98
40–49	·77	1·06	1·44
50–59	·93	1·61	2·59
60–69	1·37	2·16	5·58

APPLICATION OF ACTUARIAL PRINCIPLES AND DATA TO THE VARIOUS CLASSES OF SOCIETIES.

Sickness Benefits.

If the rate of sickness were uniform at all periods of life, friendly society finance would be a very simple matter, since societies might be conveniently carried on upon the "dividing out" principle, the members being charged an annual contribution slightly in excess of what experience had shown was sufficient for the current claims, and a small sum being kept in hand to cover fluctuations from year to year. Since, however, liability to sickness increases, like the rate of mortality, with the age of the individual, it is necessary to charge an annual contribution which will be more than sufficient, in the first instance, to meet the current sickness claims, allowing such

difference to accumulate in the hands of the society against the period when the annual claims will, by reason of increasing age, exceed the annual contribution. It is obvious that to fix this contribution at a sum which shall be adequate in the long run to provide for all the claims that will arise, without, at the same time, making it unnecessarily heavy, and, further, to determine from time to time how far the accumulations in the possession of the society are sufficient for this purpose, the actuary must be in a position to estimate, within fairly narrow limits, the amount of the claims that will in future years be made by the members upon the fund. The only basis upon which such an estimate is possible is, of course, past experience, either of the particular society, or, if this is not available or is insufficient, then of other societies similarly constituted.

The application of actuarial principles to the financial operations of friendly societies is, however, a problem of greater difficulty than in the case of life assurance; in the first place, because the two elements of sickness and mortality are involved; and, secondly, from the more frequent deviation of the sickness experience, from one cause or another, from the average type. Hence, while as regards both the mathematical theory and the data to which it can be applied, the science of life contingencies may be now said to be nearly perfect, the extension of these principles to the work of friendly societies is still in its infancy.

Causes modifying Sickness Rates.

The possibility of applying the principle of averages upon which actuarial science is based to sickness assurances, is a question that can only be decided by experience. In the case of life assurance, various reasons have facilitated this process. The fact to be dealt with, that is to say, the rate of mortality prevailing amongst a given class of lives, is well defined, and depends mainly upon the one factor of age, because although there may be disturbing causes, such as occupation, locality and the duration of the assurances, these are of minor importance, and it has been possible in practice very largely to disregard them. In the case of the average friendly society, however, we have to deal with the further element of sickness, which is much less easily defined than that of mortality. It may have, for instance, a merely relative signification depending on the nature of the occupation: sickness which would permanently incapacitate the sufferer from following certain callings, being, as regards other occupations, of little moment. Especially is this the case at advanced ages, where the line between true sickness and mere

incapacity to follow occupation is not easily drawn. The rate of sickness, as well as the actual cost of the allowances, may, moreover, be very materially modified by the rules of the individual society; by reductions in the allowances in continued sickness or their final cessation after a definite term; or by the length of the period for which a member must be off the books before being free again to claim full benefit; while there is the still further element of imposition to which societies granting sickness benefits, especially the large centralized societies, must necessarily be open.

It may be pointed out, however, that the application of actuarial principles to friendly societies does not necessarily depend on the possibility of constructing tables of sickness which shall accurately represent rates prevailing throughout the country, or that shall be generally applicable to all societies. Where a society has been actually in operation for any length of time, even though its experience may differ very widely from the average, so long as that experience has been uniform, the actuary may find himself in possession of sufficient data from the actual records of the society upon which to base estimates of future results. We have, therefore, to consider two questions: in the first place, the possibility of constructing tables of sickness which should be generally applicable; and, secondly, the extent to which past experience of any given society may be expected to reproduce itself in the future. Both of these points will be considered later, but it may be stated here that experience has undoubtedly shown that although the causes influencing the rates of sickness in a given body of persons are more numerous and more effective than in the case of mortality rates, nevertheless, whenever a group of persons or of societies is sufficiently homogeneous, the principle of averages will be found to apply as much in respect of sickness as for deaths.

Collection of Data.

The first important step in the construction of sickness tables, whether of a single society or a large group of societies, is the collection of data, and, in order that this may be trustworthy, accurate records of the experience of the society must be kept, the ages of the members under observation in any given year must be known, and the amount of sickness experienced by each carefully tabulated.

The forms and books to be employed for this purpose will, of course, vary to some extent with the magnitude and nature of the society and with the object for which the data is to be employed.

If this object is to form part of a large general experience, to include societies of various classes as to benefits and rules, forms must be used which will be universally applicable, and which, moreover, will give full information on many points which may be taken for granted where the enquiry refers to a single society only or to a group of perfectly similar institutions. Now that the quinquennial returns of sickness and mortality have been discontinued, it is these latter cases only that are in question. Passing over the general register of members and of the cash transactions of a society, it is necessary that a special register should be kept of all the sickness claims made by the members in each year, that these may be readily tabulated and analyzed when required. This register may conveniently be in the following form, which is only slightly modified from that recommended by the "Actuarial Commission" (Report 1876). The date of the commencement of sickness will enable an analysis to be made, as provided for in the "Total" columns, but it is desirable that a distinctive mark should show whether each sickness payment is at the full or reduced rate.

Register of Sickness, 188 .

Name	Date of Commencement of Sickness	Days Sick, and Amounts Paid in Week ending					Total					
		January 7		January 14		&c.	Time			Amount		
		Days	Amount	Days	Amount	&c.	1st 6 Months	2nd 6 Months	After	1st 6 Months	2nd 6 Months	After
						&c.						

It is desirable that a ledger account should be kept of every member's contributions and benefits. The form of this will vary with the class of society, but it should contain at least the following particulars :—

Name, Occupation, &c.....

		Allowances	Contributions
Entered.....		Sickness	.....
Became free		Superannuation	.....
Date of birth		Death	.....
Age (n. b. d.) at entry		Wife's death	.....

Date	Contributions Received						Date	Allowances Paid			
	Sick.	Super-ann.	Death	Mangt.	Fines	Total		Sickness or Superann.			Other Payments
								Weeks	At	Amount	

In estimating the ages of the members for the purpose of summarizing the experience, it will be found most convenient to adopt such a system as will make the assumed years of age coincident with the calendar year. This may be done by making assumptions similar to those in the Institute of Actuaries' Mortality Experience, namely, that the lives enter on the average in the middle of the calendar year and in the middle of the year of age, attaining their current age at entry (next birthday) at the close of the year of entry. The following plan, however, will be found on the whole more convenient. Let the particulars from the sickness register be posted annually on to cards of the following form, one of which should be prepared for every member entering the society:

SOCIETY.

No._____ Occupation_____

Name_____

Date of Birth, 3 May 1848.

„ *Entry*, 7 August 1879.

„ *Freedom*, 7 February 1880.

„ *Exit*, 15 September 1884.

Mode of Exit—Withdrawal.

Year	Nearest Age on 1 January	Sickness					
		1st 6 Months		2nd 6 Months		After 12 Months	
		Weeks	Days	Weeks	Days	Weeks	Days
1880	32*	...	...	...	...	...	...
1881	33	1	4	...	...	...	...
1882	34	...	...	...	...	...	...
1883	35	26	...	5	3	...	...
1884	36*	...	...	...	...	...	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The ages given are those at the nearest birthday to 1 January in each year. In the above instance, the ages opposite the years 1880 and 1884 (the years of freedom and withdrawal) are marked with an asterisk to indicate that the member was exposed to risk only for a fraction of those years.

These cards, which may be made to contain a record of 5 or 10 years' sickness, may be annually or quinquennially sorted and summarized, according to the ages of the lives under observation, the total number under observation at each age and the sickness and deaths experienced being then scheduled. If this summary be made separately for various ages at entry, it will be possible, where desired, to investigate the effect of the duration of the assurances upon the rates of sickness or mortality, while, combining the facts for all ages at entry, we obtain the materials for constructing tables of sickness and mortality in their usual form. The same assumptions should be made in respect to the numbers at risk both for sickness and deaths, that is to say, any life dying in a given year should be considered at risk for the whole of the year. If this is not done we obtain a function which may be termed, by analogy, the "central sickness rate", or force of

sickness in the middle of the year. There is, of course, no objection on principle to the employment of the functions of the force of mortality and force of sickness. It will be found, however, much more convenient to use the rates of mortality and sickness as usually understood.

The numbers at risk at each age, the deaths, and the number of weeks' sickness experienced, having been thus obtained, the ratio of the latter numbers to the former will give the probability of death within the year, usually termed the rate of mortality, and the average number of weeks' sickness, which may similarly be termed the rate of sickness, experienced by each individual.

Adjustment of Tables.

Before these facts can be conveniently employed in the construction of monetary tables, it is necessary that they should undergo a process of adjustment, or, as some prefer to term it, "graduation", in order that the irregularities incident to all observations into which the element of probability enters (however numerous the facts) may be removed. Much has been written at various times on the subject of the adjustment of mortality tables, the papers of Mr. Sprague, Mr. Woolhouse, and Mr. J. A. Higham, in the *Journal of the Institute of Actuaries*, being especially valuable. Mr. Sprague, who advocates the graphic method of adjustment, has so completely described and illustrated his system that it will be sufficient to refer the reader to his original papers.

Mr. Woolhouse, who was the first to apply the theory of finite differences to the problem of the graduation of mortality tables, and Mr. Higham, who has improved upon Mr. Woolhouse's methods and formulas, both take exception to the graphic method on the ground of the possible variety in the results that may be obtained by different operators, and upon the fact that it presupposes a considerable degree of skill and judgment in the person employing it. Without attempting to determine generally between the merits of the two systems, it may be said, that while the graphic method is, in some special cases perhaps, the only method available, on the other hand, for the adjustment of ordinary mortality and sickness tables, the use of formulas, such as Mr. Woolhouse's, Mr. Higham's, or that given below, is not open to any practical objection, and has an undoubted advantage in respect of the ease and rapidity with which they can be applied, and the certainty of the results attained. The formulas which Mr. Higham has produced give very admirable results; to give completeness, however, to the discussion of this portion of the

subject, I will add here a new formula, which I believe superior both in smoothness and simplicity to any yet given.

In this formula, if the graduated value of the function be represented by u'_x and the ungraduated by u_x,

$$u'_x = \frac{1}{120}\{24u_x + 22(u_{x-1}+u_{x+1}) + 17(u_{x-2}+u_{x+2}) + 10(u_{x-3}+u_{x+3}) + 4(u_{x-4}+u_{x+4}) - 2(u_{x-6}+u_{x+6}) - 2(u_{x-7}+u_{x+7}) - (u_{x-8}+u_{x+8})\}.*$$

Its method of application in practice is as follows. Supposing the graduated table is to commence at age 18, the unadjusted table should be brought down to age 10 by inserting constant values for the sickness and mortality rates from 10 to 18 (these may, in general, be equal to the average values of these functions at the lowest group of ages observed). At the other extremity of the table, if, as usually happens, the facts at the older ages are very small or are not reliable, it will be necessary to make some arbitrary adjustment, by inserting the rates from some suitable standard table, or in the case of the sickness by assuming it to become chronic above a selected age, remembering that the highest rate of sickness at any age will be $52 \cdot \frac{1+p_x}{2}$ weeks per annum. The point at which this should be done must, of course, depend entirely upon the judgment of the actuary as to the character of the data. It is to be noted that where the sickness is assumed to be chronic, the rates for the earlier periods of sickness must be taken as zero. For example, if, at age 85 and upwards, the rates of sickness are taken at their maximum amount, $26(1+p_x)$, then at age 86 and upwards the rate for the 1st and 2nd 6 months of illness must be taken as zero, and similarly the rates for the 2nd 12 months at age 87 and upwards.

Having in this manner completed the whole series of rates to be adjusted, the formula above given may be thus applied:

* This formula is not quite true to second differences, differing from the true value of u_x by one-twelfth of the central second difference. The error is, however, insignificant, being not greater in value than that due to the neglect of fourth differences in all such formulas. The error might be corrected by taking $\cdot 08u_x$ instead of $\frac{u_x}{12}$ in column (2), and by summing in 5's in columns (6), (7), and (8) (see next page); but there would be no practical advantage as regards accuracy, while the graduation would be less smooth.

1. Divide the rates of sickness or mortality by 12, cutting down the decimals to leave two significant figures at the young ages.
2. Sum these results in 3's, putting the result opposite the middle age.
3. From the last result subtract the sum of the two numbers in col. (2) at ages two years older and younger.
4. Sum the last result in 4's.
5. „ „ 5's, cutting down the last figure.
6. „ „ 6's, which will give the finally adjusted rates.

The form of the work will be as in the following table:

TABLE 5.—*Application of Graduation Formula.*

Age x	$\frac{1}{12}u_x = w_x$	Sum in 3's	$w_{x-2} + w_{x+2}$	(3)—(4)	Sum in 4's	$\frac{1}{10}$ Sum in 5's	Sum in 6's (Adjusted Value)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
10	·068	...	...	...	...	...	...
1	68	204	...	...	...	...	...
2	68	204	136	68	...	...	...
3	68	204	136	68	272	...	...
4	68	204	136	68	272	...	...
5	68	204	136	68	272	136	...
6	68	204	136	68	276	136	...
7	68	204	136	68	270	136	...
8	68	204	132	72	272	135	·808
9	68	200	138	62	270	133	·804
20	64	202	132	70	264	132	·801
1	70	198	132	66	257	132	·800
2	64	198	132	66	260	133	·800
3	64	196	141	55	270	135	·800
4	68	203	130	73	282	135	·797
5	71	205	129	76	277	133	·791
6	66	202	124	78	260	129	·782
7	65	187	137	50	237	126	·773
8	56	187	131	56	236	124	...
9	66	187	134	53	245	126	...
30	65	200	123	77	261	...	...
1	69	201	142	59	276	...	...
2	67	212	140	72	...	...	...
3	76	218	150	68	...	...	...
4	75	222	...	...	...	...	...
5	81	...	...	...	...	...	...
6	...	...	...	...	...	...	...

Upon the basis of the graduated rates of mortality and sickness thus obtained a table corresponding to the usual form of a mortality table should be constructed as follows:

Age	Number living at Age x	Dying between x and $x+1$	Number of Weeks' Sickness experienced by l_x Persons		
			1st 6 Months	2nd 6 Months	Remainder of Sickness
x	$=l_x$	$=d_x$	$=lz_x^1$	$=lz_x^2$	$=lz_x^{3+}$

From these values will then be computed the values for the ordinary form of commutation table, having, however, additional columns for the computation of sickness benefits; such values being calculated according to the formulas given below. The usual function N_x may be usefully replaced by the continuous function $\overline{N}_x = N_x + \frac{1}{2}D_x$, since the continuous values of annuities and assurances are those almost invariably required for friendly society calculations. The values of $\overline{A}_x$ and $\overline{P}_x$ may, of course, be calculated from those of $\overline{a}_x$ by means of Messrs. Rothery and Ryan's *Premium Conversion Tables* now in the press.

By means of these commutation tables the value of sickness benefits ceasing at certain ages and replaced by superannuation or deferred annuities, may readily be calculated—a point of considerable importance, since at the old ages there is a great lack of accurate information as to the rates of sickness prevailing; and in the great majority of cases, even where pensions are not provided for by the rules of a society, it becomes necessary to treat the sickness above a given age (for example, 70, 75, or 80) as though it were a pension at the lowest rate of pay, assuming that every member would be constantly upon the funds after these ages were attained. The value of the annuities and the funeral benefits being also given by such tables, there will thus be all the elements requisite to construct a scale of the contributions or to value the liabilities of a society.

Notation and Formulas. It will be desirable to present a brief system of notation for sickness benefits, based, as far as possible, upon that now employed for life contingencies.

z_x { = average number of weeks' sickness experienced by each individual between ages x and $x+1$.
= rate of sickness at age x.

z_x^1 = „ „ during 1st 6 months of sickness.
z_x^2 = „ „ „ 2nd „ „
z_x^{2+} = „ „ after 1st „ „
z_x^{3+} = „ „ „ 2nd „ „

lz_x { = number of weeks' sickness by l_x persons between ages x and $x+1$.

lz_x^1, lz_x^2, lz_x^{2+}, &c. = do. { during 1st 6 months of sickness. / „ 2nd „ „ / „ 2nd „ „ and subsequently. / &c. &c. &c.

$H_x = v^{x+\frac{1}{2}} lz_x$
$H_x^1 = v^{x+\frac{1}{2}} lz_x^1$
$H_x^{2+} = v^{x+\frac{1}{2}} lz_x^{2+}$
&c. &c.
} Forming the quantities in the commutation table for sickness benefits, answering to the D_x and C_x columns for annuities and assurances respectively.

$K_x = H_x + H_{x+1} + H_{x+2} + \&c.$
$K_x^1 = H_x^1 + H_{x+1}^1 + H_{x+2}^1 + \&c.$
$K_x^2 = H_x^2 + H_{x+1}^2 + H_{x+2}^2 + \&c.$
} Forming the quantities in the commutation table for sickness benefits answering to the N_x and M_x columns for annuities and assurances respectively.

$s_x = \frac{K_x}{D_x} =$ { value of sickness benefit of £1 per week throughout life.

$s_x^1 = \frac{K_x^1}{D_x} =$ { value of sickness benefit of £1 per week during 1st 6 months of illness.

$s_x^2 = \frac{K_x^2}{D_x} =$ { value of sickness benefit of £1 per week during 2nd 6 months of illness.

$s_x^{3+} = \frac{K_x^{3+}}{D_x} =$ { value of sickness benefit of £1 per week after 2nd 6 months of illness.

$_{70}s_x$; $_{75}s_x$; $_{80}s_x$
$_{70}s_x^1$; $_{75}s_x^1$; $_{80}s_x^1$
$_{70}s_x^2$; $_{75}s_x^2$; $_{80}s_x^2$
&c.
} = value of similar benefits ceasing at ages 70, 75, and 80 respectively.

$\left.\begin{matrix} P s_x\,;\ P s_x^1\,;\ P s_x^2\,;\ \&c. \\ P_{70} s_x\,;\ P_{70} s_x^1\,;\ P_{70} s_x^2\,;\ \&c. \end{matrix}\right\}$ = annual premiums payable throughout life for the corresponding benefits to s_x, $_{70}s_x$, &c.

$_{70}P s_x\,;\ _{70}P s_x^1\,;\ _{70}P s_x^2\,;$ &c. = do., premiums and benefits ceasing at age 70.

$\left.\begin{matrix} \overline{P} s_x, \text{ \&c.} \\ \overline{P}_{70} s_x, \text{\&c.} \\ {}_{70}\overline{P} s_x, \text{ \&c.} \end{matrix}\right\}$ = corresponding annual premiums when paid continuously through the year (say by weekly or monthly instalments).

As an example of this notation, we may express the single and annual premiums for a sickness benefit of £1 per week, with reduction to half and quarter-pay after 6 and 12 months respectively, replaced after age 70 by a pension of quarter-pay.

Value of sickness benefit of
£1 per week 1st 6 months.
10*s.* „ 2nd „
5*s.* „ thereafter,
to age 70, with pension thereafter, of 5*s.* per week for life.

$$= {}_{70}s_x^1 + \tfrac{1}{2}\,{}_{70}s_x^2 + \tfrac{1}{4}\,{}_{70}s_x^{3+} + 13\,{}_{70-x|}\bar{a}_x .$$

$$= \frac{K_x^1 - K_{70}^1 + \frac{1}{2}(K_x^2 - K_{70}^2) + \frac{1}{4}(K_x^{3+} - K_{70}^{3+}) + 13\overline{N}_{70}}{D_x}$$

Annual premium for same, payable weekly or monthly } Denominator $= \overline{N}_x$

or if ceasing at age 70 . . . Do. $= \overline{N}_x - \overline{N}_{70}$.

Sickness Tables—Existing Data.

Highland Society's Tables.

The earliest attempt to construct tables of sickness, based upon the collected experience of friendly societies, was that made by the Highland Society in 1820, the results of which were published in 1824. The table is historically interesting, and merits notice principally from the fact that it was largely employed (with unfortunate results) between that date and its withdrawal in 1851, when its unsafe character had become apparent to its authors. The data were voluntarily contributed by 73 societies (out of a much larger number applied to) from the various counties of Scotland, and were sufficiently extensive (up to about 70 years of age) to afford a fair basis for a sickness table, the number of years of life observed being over 104,000.

It was scarcely to be expected that a first attempt to produce a table that should be generally applicable would prove entirely successful, and subsequent experience speedily showed that the table gave far too favourable a view of the sickness prevailing on the average among the members of friendly societies. The cause of the very low rate of sickness shown has been much discussed by various authorities. It has been variously attributed to the superior healthiness of the districts from which the returns were collected, the inaccuracy of the data, and to the fact that many of the members of the contributing societies did not claim in case of illness (though an attempt appears to have been made to correct the returns in this respect). In addition to these possible explanations however, the following considerations seem to me to go far towards reconciling the results with those of later investigations. In the first place, the returns were purely voluntary, and were furnished by a small proportion only of the societies applied to; many societies strongly objected to the enquiry, many were indifferent, and still more had not kept their books in a form to afford the desired information. Considering the marked differences which will be shown to exist between the sickness experiences of individual societies, it does not appear unlikely that a principle of selection would work in the production of such voluntary returns by which societies whose experience had been favourable would be more likely to respond than where the sickness rates had been above the average, and, further, that it would in general be the best-managed societies alone who would be in possession of the facts required.

The rates of sickness as given are, moreover, not fairly represented in the resulting table, owing to the very unhappy method of adjustment employed. The original returns gave the numbers at risk and sickness in decennial groups of ages, 20–30, 30–40, &c. To obtain the sickness rates at individual ages, these numbers might have been adjusted by some method similar to that employed by Mr. Milne in dealing with the Carlisle Table, and the numbers living and sickness experienced at each age deduced, or the rate given by the decennial groups might have been taken as that for the middle age of the group. The method actually adopted was to make the average of the sickness rates for ages 21, 22 . . . 30 equal to the rate deduced from the numbers exposed to risk between ages 20 and 30. This method would only have been sound where the exposures were the same at each age in the group, and at the older ages it greatly under-

estimated the true rate, to say nothing of the fact that the rates entered against the ages 21, 22, 23, &c., were those properly belonging to ages 20, 21, 22, &c. This erroneous method of adjustment reduced the observed rates at the older ages by fully 20 per-cent. Finally, all entrants and withdrawals were estimated at risk for an entire year, materially affecting the rates at the younger ages. I have made an estimate of the extent to which the final sickness rates were affected by these causes, and in Table 9 the third column gives what I believe to be a fair approximation to the original facts.

The monetary tables were affected by another circumstance. No investigation was made into the mortality experienced by the societies, and in constructing the tables a mean between the Northampton, Carlisle, and Swedish rates of mortality was adopted. The inclusion of the first-named table probably resulted in the rate of mortality being over-estimated, and, taken into conjunction with the extremely low sickness rates observed, rendered the resulting tables very unsafe.

Mr. Ansell's Tables.

In 1835 Mr. Charles Ansell published his treatise on friendly societies, containing tables based on the experience of various societies for the five years 1823–27, embracing about 5,000 members and about 24,000 years of life. The rates of sickness observed were considerably higher than in the Highland Society Tables, and at the early ages, up to age 35, they differed but little from those of the Manchester Unity Tables (1866–70), and from 35 to 65 were generally lower than the latter. Mr. Ansell did not attempt to carry the sickness rates beyond age 70, but for a considerable period prior to that age the data were not sufficient to furnish trustworthy results. The rates were obtained from the numbers at risk, the deaths and the sickness in quinquennial groups of ages. Above age 70 the Northampton rates of mortality were employed, a somewhat doubtful step. Although the table was a great improvement upon that of the Highland Society, and came much nearer to the experience of average friendly societies, the number of societies constituting the experience was too small for a table based upon them to be generally applicable, and subsequent experience has shown that the tables were unsafe for the large majority of societies.

As in the case of the Highland Tables, no attempt was made to analyze the sickness in respect of duration, an important enquiry in view of the nearly universal plan of reducing the allowances in prolonged sickness.

Mr. Neison's Tables (1836-40).

In 1845 Mr. Neison published extensive tables, based upon an analysis of the Government returns from 1836 to 1840. The data left nothing to be desired as regards extent, embracing 1,147,000 years of life, with a considerable experience at the older ages. In these tables the division of the experience into rural, town, and city districts, which is common to all subsequent investigations, was, it is believed, first made. Mr. Neison also investigated the rates of sickness and mortality prevailing in various occupations, and he separated the Scotch experience, in which the sickness rates up to age 50 were considerably lower than those of English experience, although the rates of mortality were throughout heavier.

The principal points to be noticed in these tables are the very light rates of mortality recorded, and the heavy rates of sickness, especially above age 65, the latter feature forming a strong contrast, not only to the earlier tables of Mr. Ansell and the Highland Society, but also to those deduced by Mr. Finlaison from the Government returns 10 years later. The divergences between these tables illustrate the difficulty of distinguishing between sickness and superannuation at the older ages. While, on the one hand, Mr. Finlaison's wish to exclude all cases of superannuation no doubt resulted in the exclusion of a considerable portion of chronic sickness, and rendered his tables practically valueless for the average friendly society, on the other hand, it may be shown, I think, from careful examination of the figures upon which Mr. Neison's Table is based, that in many cases allowances that were purely superannuation and not in any sense sickness payments were included. If reference is made to the summary of the facts of Mr. Neison's experience, as given in his *Vital Statistics*, it will be seen that in each of the three districts —rural, town, and city—the quantity of sickness observed makes a sudden bound, in the case of the two former at the commencement of the group 65–69; and in the latter at 70–74. This increase is so sudden that it evidently points, not to any real increase of the sickness at these ages, but to the practice of many of the societies included in Mr. Neison's returns of practically pensioning their members at these ages whether sick or not. Hence, in societies making allowances only in cases of actual sickness, they are unnecessarily stringent. The very light mortality brought out in this investigation resulted, together with the heavy sickness rates, in a very high value for the single and annual sickness premiums, the latter exceeding Mr. Ansell's values by 25 to 30 per-cent.

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From whatever cause, it would seem probable that the mortality was under-estimated in Mr. Neison's Table, in the rural districts especially, where the rates are lower than those given by the English Life Table for ages five years younger. Mr. J. A. Higham has suggested that this may have resulted from the immense number of insurances that were discontinued, not as in the case of insurance companies by the healthy and prudent, but by the unhealthy and reckless. This explanation, however, seems scarcely sufficient, and is contrary to later experience. It is possible that in some cases, where societies insured for sickness only, some of the deaths may have been returned as discontinuances, but no doubt the explanation is to be found mainly in the uncertainty as to the facts, not only as regards the actual number of deaths recorded, but especially in respect of the ages returned. The ages of the lives beyond 60 or 70 must have been in many cases a matter of guess-work, and beyond age 80 the recorded rate of mortality actually diminishes instead of increasing, which makes it extremely probable that here at least the ages given were altogether erroneous and, no doubt, greatly exaggerated. It may be of interest to give here a comparison of the rates of mortality in Mr. Neison's rural experience with those deduced from the experience of a large rural society.

TABLE 6.—*Rural Mortality Experience.*

Ages	Mortality per-cent			
	Essex Provident Society, 1876-80		Neison's Rural Districts, 1836-40	
	Rough	Adjusted	General	Labourers
(1)	(2)	(3)	(4)	(5)
15–19	·00	·52	·73	·50
20–24	·65	·59	·73	·64
25–29	·34	·68	·73	·57
30–34	·93	·79	·69	·57
35–39	·83	·92	·77	·61
40–44	1·12	1·08	·82	·75
45–49	1·45	1·28	·97	·84
50–54	1·34	1·55	1·35	1·11
55–59	2·02	1·97	1·82	1·49
60–64	2·71	2·74	2·36	1·78
65–69	4·33	4·21	3·65	3·17
70–74	6·92	6·80	6·07	5·25
75–79	10·37	10·98	7·53	6·86

The differences in the rates in columns (3) and (5) are instructive, but it is unnecessary to discuss them at length. The fact that the mortality in the various districts observed by

Messrs. Neison and Finlaison differ so greatly, although both were deduced from a similar class of returns, throws a doubt upon the trustworthiness of the facts in each case.

Mr. Finlaison's Table (1846-50).

The next tables of importance are those of Mr. Finlaison, already alluded to. These were based on the Government returns from 1846 to 1850. As already stated, all cases of superannuation were excluded from this inquiry. In Mr. Finlaison's own words (see Parliamentary Paper, 12 August 1854, page 7), "Nothing but sickness in the true sense of the "word, that is, sickness incapacitating from labour and requiring "constant medical treatment, and of limited duration, as dis-"tinguished from chronic ailment and mere decrepitude, was "considered to be sickness. For instance, slight paralysis, blind-"ness, mental disorder, or senile infirmity was not included." Had these cases been separately treated the results would have been both interesting and valuable, but a great quantity of what, in the majority of friendly societies, constitutes sickness at the older ages being entirely disregarded, Mr. Finlaison's Tables are necessarily unsuited to such societies. The sickness rates in Mr. Finlaison's Tables are comparatively heavy at the younger ages, where the exclusion of chronic sickness would have but little effect. Up to age 40, for example, the rates are heavier than those of the Manchester Unity, while above 50 they are very much lower, nearly corresponding to the rates at five years younger. Below age 50, moreover, the rates in rural, town, and city districts were very nearly identical, differing in this respect from the results of Mr. Neison's investigation. It should be stated, however, that the city districts in Mr. Finlaison's classification included only populations of over 65,000, as against populations of 30,000 in Mr. Neison's.

Mr. Finlaison divided the experience, with reference to the nature of the occupation of the lives observed, into light, heavy, and general labour, including a special investigation into the sickness rates prevailing amongst miners, mariners, railway servants, and painters. His results went to show that, so far as that class of sickness is concerned with which he dealt,—that is to say, chiefly temporary sickness,—the character of the labour is of more importance than the locality from which the lives were drawn, or the density of the population. He states that the only practical difference in the distribution of sickness that could be discovered was seen to turn on the amount of physical force called out by the occupation of the contributors. It is doubtful, however, whether this theory could be substantiated, if the chronic

sickness had been included. The sickness prevailing amongst the female members of the societies was also dealt with and the tables were accompanied by a complete system of monetary values, based on a rate of interest of 3½ per-cent.

Mr. Ratcliffe's Manchester Unity Tables.

Almost simultaneously with Mr. Finlaison's report, the Manchester Unity of Odd Fellows made an investigation into the mortality and sickness experienced in their society for the three years 1846 to 1848. A similar investigation was made for the five years 1856 to 1860, and again 10 years later for the period 1866 to 1870. The earlier tables were doubtless much less accurate than the last ones, which have for some years held the position of standard sickness tables.*

From the fact that the later returns, 1866 to 1870, are the third series tabulated by the Manchester Unity, there is every reason to believe that the data are, in the main, trustworthy, and they have probably afforded the best series of observations with which we have at present been furnished. The extreme difficulty of obtaining an accurate record of facts relating to sickness experience from average friendly societies is well known to all actuaries, and there can be no doubt that the earlier returns furnished to the Government in many cases abounded in errors, but it is probable that greater reliance may be placed upon returns obtained from the branches of an order such as the Manchester Unity, especially where these have not been made for the first time.

The most important feature of the Manchester Unity experience is the separation of the sickness claims made during the 1st, 2nd, 3rd, and 4th periods of six months from the commencement of illness. This analysis, which was common to the two last investigations—1856–60 and 1866–70—was only carried out approximately, however, owing to the absence of complete information as to the date at which the cases of prolonged sickness claim had commenced. So far as can be ascertained this date was taken in practice as 1 January 1856 and 1866 respectively, whenever the lives observed were sick at that date. The obvious effect of this assumption is to exaggerate the proportion of sickness occurring in the first six months of illness, and to diminish in like

*In the earlier returns numerous inaccuracies were apparent—especially is this the case with those of 1856 to 1860. For example, on reference to Table 5 (p. 17), the numbers at risk at age 52 appear to have been put down at 7,443 instead of 6,443, and as a consequence the rates of death and sickness from 51 to 55 are under-estimated about 3 per-cent. A similar error occurs in Table 6 (p. 19), at age 50, where the numbers 6,180 should clearly read 5,180. Even as the figures stand the rate of sickness given for this quinquennium is erroneous, the same being the case also from 61 to 65, where the rate should be 5·950 instead of 4·625. These and other errors have affected the final Table, although the effect on the financial results may have been small.

manner that relating to subsequent periods; and the general experience of actuaries will, I think, confirm the impression that the Manchester Unity Tables do, as a matter of fact, considerably under-estimate the relative amount of prolonged sickness in most societies. The financial effect of the error is of course upon the safe side.

A.O.F. Experience (1871-75).

The Ancient Order of Foresters have also published the results of an investigation into their sickness and mortality experience for the five years 1871–5, made by Mr. Neison, Junior, whose report and analysis of the experience is very elaborate, and contains much interesting information. The experience was very extensive, comprising 1,300,000 years of life; but there are manifest traces of the same exaggeration of the ages in advanced life which has been noticed in the earlier data of Mr. Neison, Senior, and which would suggest that at present at least, the facts as to the mortality and sickness of the Ancient Order of Foresters have not been obtained with the same accuracy as in the Odd Fellows' experience. The distribution of the experience into rural, town, and city, differed considerably from the Manchester Unity, as the following table will show:

TABLE 7.—*Distribution of Data.*

	Odd Fellows (1866-1870)	Foresters (1871-1875)
Rural Districts . . .	222 per M.	324 per M.
Town „ . . .	513 „	291 „
City „ . . .	265 „	385 „
Total Experience .	1,000	1,000

Thus the Manchester Unity prevails principally in the smaller towns, the Ancient Order of Foresters in the city and rural districts.

The same difficulty occurred in determining the exact proportion of sickness falling in the first six months of illness as with the Manchester Unity experience. Mr. Neison, however, by means of certain supplementary returns and certain assumptions as to cases of continuous sickness, succeeded in partially overcoming this difficulty.

The aggregate sickness rate in the Foresters' experience is a trifle higher than in the Manchester Unity, but the proportion of prolonged sickness is considerably greater.

Correction of the Sickness Rates in the various periods.

It is worth while to point out that the want of complete information as to the duration of sickness in the Manchester Unity and the Foresters' (original) returns, might have been practically overcome by a simple correction.

Since all sickness recorded in the first half-year of the quinquennium was treated as belonging to the first six months of sickness, it will be seen that, speaking generally, sickness returned as belonging to the second and subsequent periods of sickness would comprise only such as fell in the last four-and-a-half years of the quinquennium; that returned as of the third and subsequent periods would comprise only such as fell in the last four years.

Hence, if we call the recorded rates of sickness in the three periods a, b, c, and the true rates z^1, z^2, z^{3+}, and assume the exposures to be equally distributed through the five years, we should have

$$z^{3+} = \frac{10}{8}c\,; \quad z^{2+} = \frac{10}{9}(b+c)$$

$$\therefore \quad z^2 = \frac{10}{9}(b+c) - \frac{10}{8}c$$

$$z^1 = (a+b+c) - \frac{10}{9}(b+c)$$

As a matter of fact, the exposures would not be equally distributed; although the exact distribution would have been easily ascertainable. If we assume that 7 per-cent of the experience belonged to the first half-year, and 14·7 per-cent to the first year (which will probably be near the truth), the following table will show the resulting corrections to the rates of sickness in the various periods:

TABLE 8.—*Manchester Unity Experience* (1866–70).

Ages	Recorded Rates of Sickness			Corrected Rates		
	1st 6 Months	2nd 6 Months	After 12 Months	1st 6 Months	2nd 6 Months	After 12 Months
22	·683	·039	·019	·679	·040	·022
27	·711	·049	·046	·704	·048	·054
32	·773	·068	·085	·761	·065	·100
37	·847	·080	·134	·831	·073	·157
42	·959	·114	·204	·935	·103	·239
47	1·142	·168	·360	1·102	·146	·422
52	1·417	·246	·579	1·353	·209	·678
57	1·817	·376	·953	1·717	·312	1·117
62	2·405	·628	1·776	2·224	·503	2·082
67	3·234	1·074	3·248	2·910	·844	3·803
72	3·910	1·675	6·516	3·293	1·178	7·630
77	4·130	2·281	10·691	3·154	1·429	12·519

It is true the correction made does not show the full extent of the error, as it takes no account of those cases of continued sickness commencing in the first half-year of the quinquennium, in which the member (on account of recent claims upon the fund) was not entitled to draw full pay for the full period of six months. These cases, however, would not be very numerous.

Comparison of various Sickness Tables.

The following tables give in a concise form the means of comparison of the rates of sickness and mortality in the various tables that have been referred to.

TABLE 9.—*Comparative Sickness Rates.*

Ages	Highland Society		Ansell, 1823-27	Neison, 1836-40	Finlaison, 1846-50	Manchester Unity			A.O.F. 1871-75	Ages
	As Given	As Estimated				1846-48	1856-60	1866-70		
20–24	·576	·680	·785	·857	·985	·678	·829	·754	·819	20–24
25–29	·596	·680	·825	·890	·985	·758	·820	·806	·853	25–29
30–34	·641	·710	·893	·917	·971	·837	·858	·928	·968	30–34
35–39	·702	·770	1·009	1·036	1·091	·940	1·004	1·062	1·150	35–39
40–44	·814	·880	1·195	1·273	1·227	1·179	1·239	1·261	1·373	40–44
45–49	1·108	1·190	1·475	1·634	1·437	1·515	1·545	1·636	1·707	45–49
50–54	1·541	1·730	1·890	2·181	1·732	2·030	2·011	2·222	2·265	50–54
55–59	2·018	2·380	2·586	3·048	2·177	3·206	3·025	3·047	3·210	55–59
60–64	2·736	3·980	3·991	4·745	3·118	5·059	4·638	4·715	4·594	60–64
65–69	6·600	7·220	7·471	10·012	4·622	6·495	7·165	7·237	7·971	65–69
70–74	...	...	...	16·529	7·668	12·151	12·056	12·062	11·999	70–74
75–79	...	...	...	25·064	11·056	16·716	16·637	16·873	17·662	75–79

TABLE 10.—*Comparative Rates of Mortality (Per-cent).*

Ages	High-land Society	Ansell, 1823-27	Neison, 1836-40	Finlaison, 1846-50	Manchester Unity			A.O.F., 1871-75	Ages
					1846-48	1856-60	1866-70		
20–24		·679	·667	·748	·740	·758	·643	·743	20–24
25–29		·989	·726	·729	·790	·748	·762	·729	25–29
30–34		1·143	·774	·796	·870	·834	·818	·886	30–34
35–39		1·788	·875	·893	·916	·991	·977	1·092	35–39
40–44	Not recorded.	1·663	·992	1·100	1·165	1·178	1·258	1·284	40–44
45–49		2·014	1·201	1·306	1·399	1·421	1·429	1·658	45–49
50–54		2·290	1·567	1·636	1·861	1·795	1·905	2·045	50–54
55–59		3·635	2·120	2·360	2·867	2·609	2·492	2·973	55–59
60–64		4·433	2·772	2·855	4·114	3·566	3·537	3·802	60–64
65–69		5·504	3·968	4·391	5·721	5·499	5·209	5·843	65–69
70–74		10·417	6·732	6·203	7·042	6·825	7·811	8·003	70–74
75–79		7·407	8·146	9·209	...	11·538	9·950	14·259	75–79

It will be observed that these results are quite as accordant in respect of the sickness rates as in the case of the mortality rates.

When the various circumstances under which the data were collected is taken into account, and the different methods of treatment to which the facts were subjected, their agreement appears more remarkable than their differences. If a comparison is made of the Manchester Unity (1866–70), the Ancient Order of Foresters (1871-75), and Mr. Neison's results (1836-40), the three largest series of observations, the agreement in the sickness rates up to age 65 is remarkable. If an average of the three rates be taken, in only one case do any of the figures differ by so much as 5 per-cent from this average.

Effect of "Selection" on the Sickness Rates.

It is well known that the effect of "selection", as it is termed, upon the rates of mortality prevailing among assured lives is considerable, and it would seem natural to suppose that a similar though less marked effect would be produced by the same cause upon the rates of sickness: less marked, because the selection is, on the whole, less strict, and, further, cannot be expected to eliminate early sickness claims to the same extent as early deaths. There are no data of any extent available by which this question can be determined, and the experience of individual societies no doubt differs as to this point very considerably. In Mr. Neison's Report, Ancient Order of Foresters' experience, 1871–75 (page 28), the effect of medical selection is referred to in connection with a table showing the percentage of members sick during the quinquennium out of

(*a*) Those in the courts, 1 January 1871;

(*b*) Those who became "free" during the quinquennium.

From the smaller percentages in the second class Mr. Neison draws the conclusion that "for the first few years after admission the medical selection employed is not without practical effect." It is clear, however, that since the lives entering during the quinquennium are at risk for only about half the period of those existing at its commencement, the comparison cannot legitimately be made the basis of such conclusion. While the subject is not perhaps of the first importance, the following table, comprising the analyzed experience of eight societies of an average type, may be found of interest as throwing some light upon this question. The facts are not very numerous, but are sufficiently so to indicate that recent selection does influence the sickness rates at the various ages; but to an extent which can scarcely be considered important, or necessary to be taken into account in the calculation of contributions or in the valuation of societies.

TABLE 11.—*Illustrating the effect of recent selection of Members upon the Sickness Rate.*

(Compiled from the experience of eight Societies.)

Ages	1st and 2nd Years of Assurance					3rd, 4th, and 5th Years of Assurance					6th and Subsequent Years				
	At Risk	Number of Weeks' Sickness			Cost per Member *	At Risk	Number of Weeks' Sickness			Cost per Member *	At Risk	Number of Weeks' Sickness			Cost per Member *
		1st 6 Months	2nd 6 Months	After			1st 6 Months	2nd 6 Months	After			1st 6 Months	2nd 6 Months	After	
15–19	292	151	26	...	·56	5	2	...	...	·40	...	...	...	...	...
20–24	960	489	76	30	·56	791	526	31	75	·71	87	62	...	...	·71
25–29	623	363	44	...	·62	973	653	46	6	·70	1,028	763	21	...	·75
30–34	310	183	8	...	·60	599	509	68	35	·75	1,728	1,442	149	79	·81
35–39	125	85	13	...	·72	352	205	10	7	·60	1,908	1,619	148	68	·90
40–44	14	14	7	...	1·25	112	139	14	...	1·30	1,489	1,423	131	79	1·01
45–49	...	...	...	...	...	13	12	...	...	·92	895	1,037	241	601	1·46
50–54	...	...	...	...	...	...	...	...	...	...	561	641	53	625	1·47
55–59	...	...	...	...	...	...	...	...	...	...	330	438	48	444	1·74
60–64	...	...	...	...	...	...	...	...	...	...	250	469	99	347	2·42
65–69	...	...	...	...	...	...	...	...	...	...	205	332	159	957	3·17
70–74	...	...	...	...	...	...	...	...	...	...	141	198	129	2,857	6·93
75–79	...	...	...	...	...	...	...	...	...	...	62	73	59	2,353	11·14
Totals	2,324	1,285	174	30	...	2,845	2,046	169	123	...	8,684	8,497	1,237	8,410	...

(* "Cost" estimated by taking £1 per Week for 1st 6 Months, 10*s.* per Week 2nd 6 Months, and 5*s.* thereafter.)

Valuation of Friendly Societies.

Basis of Valuation.

The object of a valuation is to estimate, as nearly as possible, the present value of future payments to be made by the society, on the one hand, in respect of the benefits assured to the members for sickness or deaths; and by the members on the other hand, in respect of the contributions to be received from them; and hence to determine what sum it is necessary that the society should have in hand to be able eventually to meet all the claims as they arise. It is, therefore, important that the assumptions made in the valuation in respect of the rates of mortality and sickness that will prevail, and not less important the rate of interest at which the funds may be expected to improve, shall be such as past experience shows may be made with safety. Such a valuation will only be useful as the future working of the society may coincide with the assumptions upon which it is based, and the first business of the actuary, therefore, will be to make himself acquainted with the actual experience of the society whose financial position he is investigating, that he may select the most suitable basis upon which to frame his estimates.

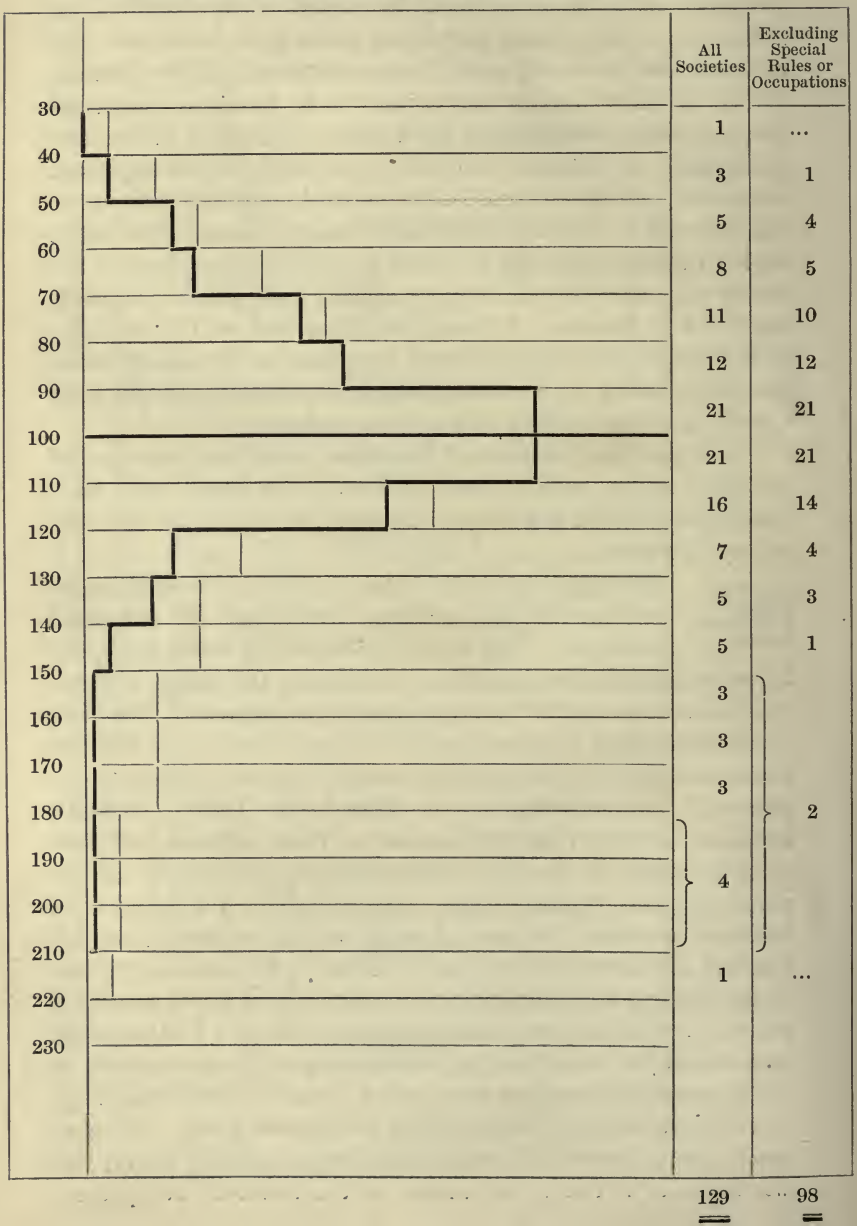
In valuing the liabilities of life offices, a uniform standard of mortality might, without much difficulty, be selected. No such general standard of sickness can, however, be adopted for friendly society valuations.

Variety in Sickness Experience of Societies.

Reference has already been made to the differences existing in the sickness experience of individual societies. With a view of illustrating this feature, the following diagram has been made, exhibiting the variety of experience found amongst 129 societies, taken promiscuously. The basis of the comparison was the ratio of the actual cost of the sickness allowances during a given period (usually five or ten years), to the estimated cost according to the Manchester Unity (1866-70) experience and the rules of the society. These ratios are exhibited as percentages on the left of the diagram, the length of the parallelograms opposite being proportional to the number of societies in which the ratio of actual cost to estimated cost fell between the various limits shown. Thus, in 21 societies the cost of the sickness was between 90 and 100 per-cent of the estimated amount, and in 21 other societies between 100 and 110 per-cent. In a number of cases the light sickness observed was traceable to the nature of the society's rules, which, as will be seen later, may, in various ways, keep down the rate of sickness claim. In other cases, where the cost of sickness had been specially heavy, this was found to be due to the nature of the members' occupations.

When these cases are excluded, there remain the 98 societies shown in the last column of the diagram, and represented by the parallelograms bounded by the heavy lines.

TABLE 12.—*Comparison of the Relative Cost of Sickness in* 129 *Societies with that shown by the Manchester Unity Table.*



Deviations from Standard Table.

The extent of data requisite to properly test the experience of a society and to give a fair idea of the sickness rates that may be expected to prevail in the future is a question closely related to that to be presently discussed—namely, the extent of membership required to give a steady experience. Usually the experience of five years' working will be found to be sufficient, provided it is of an average class, without any special features. If, however, the society is small, or if, from the nature of the members' occupations or from the rules governing the sickness allowances, the experience presents special features which render a departure from the standard tables necessary, it is desirable that the period embraced should be at least 10 years. The simplest form of exhibiting the sickness experience of a society is that of comparing the actual sickness observed among the members of different groups of ages and the estimated amount, according to any standard table that may be employed. Such a comparison will show whether the sickness is normal as to its aggregate amount, as to its distribution at various periods through life, and its division into full pay and reduced pay. Where deviation from the standard table occurs, its cause and nature must be examined. It may occasionally happen, especially in very small societies, that such deviation is accidental. It may be caused, for example, by a single member continuing sick during the period under observation, thus producing a very high average rate of sickness in that group of ages in which he is included. As to how far deviations from the average table are to be considered significant must of course depend upon their amount, relative to the magnitude of the experience. Reference should be had to the results obtained later in considering the question of sickness and mortality fluctuations.

Occasionally a simple modification of the standard table, such, for example, as an increased percentage on the sickness rates at all ages, will meet the case. Where the values of the sickness allowances by the standard table employed are available for various rates of interest, and terminating at various ages, it is possible to vary the table almost indefinitely, and thus obviate the labour of calculating a special sickness table. Thus, provided the society's mortality is well represented by the standard table, it is possible to employ any rates of sickness that can be represented by constant or percentage additions to (or deductions from) the standard rates—*e.g.*, suppose that the full-pay sickness

experienced by the society is nearly a constant of ·5 weeks per annum above the standard, and the reduced pay is well represented through life by an addition of 20 per-cent at all ages, a valuation would be first made by the standard table, the value of the reduced pay (up to the "pension" age) increased by 20 per-cent, and an addition of the value of an annuity of half the weekly allowance added to the value of the full pay as brought out by the table. When it is necessary, also, to vary the mortality rates, this may be done either by a variation in the ages, or by a constant addition to (or deduction from) the value of μ_x at all ages, which, as is known, will affect the values of the annuities and sickness benefits in precisely the same manner as a similar change in the force of interest. Where a change is thus made in the rates of mortality by a variation in the assumed ages, the sickness rates must, of course, be represented in terms of the rates at the new ages according to the standard table.

Employment of the Society's Experience.

It may prove impossible by any such simple modification to represent the observed facts. In such cases it is necessary to base our valuation estimate on the rates actually found to prevail in the society. In other words, taking the observed experience as our basis, a sickness table must be constructed to represent that experience, in the same way in which the standard table itself was formed from the larger general experience upon which it was based. Where a valuation is thus based upon the actual experience of the society, care should be taken to see that the experience has been tolerably uniform over a series of years, and that the data are sufficient to justify dependence upon the results. It is not sufficient to represent merely the average experience of a decennium, or even a longer period, unless it is known that there has been no marked change in the character of the experience during that period. Such a change may result from various causes, as, for example, from a relaxation of the management, or from the introduction of a class of lives more liable from their occupation to sickness than those previously existing in the society; or a change may even be caused by an alteration in the benefits granted, especially by an increase in the allowances for prolonged sickness. Cases may even occur in which it is impossible to employ the same standard for all the members of a society. That is to say, we cannot assume merely on the basis of the sickness experience that the rates of sickness found to prevail at the different ages through life during the period under observation will actually prevail at

these ages in the future. It may be useful to give an actual illustration of this point. The following table represents the experience of a small country society over a period of five years.

TABLE 13.—*Example of Non-Homogeneous Sickness Experience.*

Ages	At Risk	Estimated Cost of Sick Pay	Actual Cost	Actual to each 100 expected
		£	£	
15–19	355	116·0	133·5	115
20–24	303	108·0	158·4	147
25–29	231	87·8	136·7	156
30–34	196	83·3	146·6	176
35–39	154	73·5	143·3	195
40–44	102	57·2	44·6	78
45–49	47	33·3	22·9	69
40–54	46	41·5	27·0	65
55–59	37	46·4	51·9	81
60 & upwards	14	25·5	6·8	

It will be seen by a comparison of the expected cost of sickness with the actual cost, that the experience may be divided into two parts, namely, that below age 40 and that above that age. In the former group the sickness claims were more than 50 per-cent in excess of the expected amount, while in the latter they were exactly 25 per-cent below the expectation. An examination of the cause of this break in the society's experience showed that it was due to the introduction of a large number of new members following hazardous occupations. It is clear that a valuation proceeding on the assumption that the experience of the past five years in respect of the sickness rates prevailing at the various ages would be reproduced in the future, would have greatly under-estimated the future liabilities of the society in respect of the sickness allowances. The only safe method of procedure in such a case is to divide a society into two classes, namely, those subject to the normal rate of sickness, and a second class, formed of those whose occupations render them liable to excessive sickness rates. This same experience also affords an illustration of the extent to which a single member may affect the character of the experience. It will be seen that if the two groups of ages 30 to 39 be taken, the observed sickness claims in these groups exceeded the expected amount by about £133; of this excess, however, no less than £70 is due to sickness experienced by a single member.

Sickness Claims modified by Rules.

Occasionally the distribution of sickness at various ages will be considerably affected by the nature of the rules modifying or limiting the sickness benefits, and it may be of interest here to give some actual instances showing the character of these modifications. The most common case occurs where, in lieu of a reduction in the sickness rate in cases of prolonged sickness, the benefits for a certain time cease altogether. In these cases it will generally be found that, notwithstanding the fact that the society's rules do not recognize chronic sickness claim, they cannot be entirely ignored in the valuation, as although the member cannot be continuously upon the funds, he may nevertheless come on repeatedly at intervals. For example, a society granting 10*s.* a week allowance during the first six months, 5*s.* during the second, and no allowance thereafter, nevertheless, in the case of continuous sickness, must after a certain period (generally after 12 months from the cessation of the half-pay allowance) permit the members to come again upon the funds, if sick, at full pay, treating the case as a fresh case of illness. Hence, on the average, in every two years a member who is permanently invalided will be able to draw 26 weeks of full, and 26 weeks of half-pay, and this would be equivalent to a permanently reduced allowance of 3*s.* 9*d.* per week, and should be treated as such at a valuation. It will, nevertheless, be found in practice that this cessation of the sickness allowance after 12 months or some such period of claim does, as a matter of fact, very materially reduce the aggregate amount paid for sickness, especially at the older ages.

The following table represents the experience of a society in which the distribution of the sickness at various ages has been materially influenced by a rule permanently reducing the allowance after a total of 52 weeks' claim had been made, with a further reduction after a second period of 52 weeks.

TABLE 14.

Example of Effect of Permanent Reduction of Allowance.

Ages	At Risk	Estimated Cost of Sickness, M.U. Full Pay throughout	Actual Cost	Percentage
15–19	65	21	8	73
20–29	288	112	89	
30–39	414	208	94	45
40–49	369	270	155	57
50–59	184	244	105	43
60–69	79	215	484	225
70–79	23	174	200	115

A comparison of the estimated amount of the sickness claims at the various groups of ages with those actually paid will show that the effect of this rule is mostly felt during middle life. The sickness at the younger ages is nearly normal in its amount, but after about age 30 there is evidently a tendency to postpone the claims, with a view to avoiding the reduction in the allowances as old age is approached. This effect continues to about age 60, when there being probably a large number of cases in which the full period of 52 weeks' pay has been drawn, and the reduction actually effected in the allowance, the rate of claim rises very rapidly, and at the older ages is considerably above the Manchester Unity Table. This society represents a case in which the average tables would be inapplicable, and where it would be necessary to base the valuation estimate entirely upon the rates of claim that had actually been experienced at the different ages. In such cases it is clear that, with a limited experience, it must be difficult to forecast the future with any degree of certainty, there being so many circumstances which may modify the society's future experience. Consequently it is necessary to allow a considerable margin over and above the rates actually observed if our estimate is to be a safe one.

Benefits varying with duration of Membership.

There are certain cases in which the ages of the members cannot be taken as the basis of the valuation, the special rules of the society making the duration of membership a more important element than age. As a typical case may be mentioned a society in which the members are divided into classes, according to the period of membership: the first-class consisting of all who are insured under five years, the second from 5 to 10 years, the third from 10 to 15, and so on, the sickness allowance increasing each time a member is transferred to a higher class. It is here evident that the cost of the sickness claims amongst any group of members will depend rather on the duration of their membership than upon their age. And since in a small society a double classification would be impracticable, it is necessary to treat all the members as entered at one average age, and to base the observations upon years of insurance rather than years of age. The same society has a rule which is not uncommon among societies of this class, in which the funeral allowance made to a member is modified by the sickness claims that have been made upon the society. In this particular instance the amount of funeral allowance increases, like that of the sickness allowance, with the duration of membership to a maximum sum

of £10. If, however, any member had during his lifetime drawn a larger sum in special sickness allowances than the contributions he had paid to the society, the amount of his funeral benefit was reduced to £4; while, on the other hand, members who had been a sufficient time in the society and had not drawn more than a certain sum for sickness claims during the period were allowed £15 at death. It is clear that rules of this kind make the estimation of the future sickness and funeral benefits a matter of considerable complexity. The difficulty, however, may be fairly surmounted by assuming an average age at entry for each member, as proposed above, classifying them according to the duration of membership, investigating the average cost rather than the average time of sickness for various periods of membership, and in the case of the funeral claims determining the average amount of the allowance when paid for the same periods, and treating the benefit as a variable insurance. This modification of the funeral benefit, increasing it by way of bonus where the sickness claims have been small, or diminishing it where they have been excessive, is by no means uncommon in practice, and will be found generally to have a marked influence on the rates of sickness claim.

Sickness Benefit in Old Age.

In general the actual experience of any society will afford but little data upon which to estimate the sickness rates at the most advanced ages. This is a point of considerable importance at a valuation, since, although the number of members in a given society at the older ages may be very small, the value of the sickness claims that will be made at these ages will always be found to represent a considerable portion of the society's liabilities. What is true of individual societies in respect of want of information as to sickness claims in old age, is also true to some extent of all the general sickness tables. Though many of these tables profess to show the rates of sickness throughout life, the data beyond about 70 or 75, certainly beyond age 80, is quite untrustworthy. The ages of the members returned must be at these periods of life very uncertain, and no doubt are in many instances exaggerated. Even if this were not so it would be still very hazardous to assume, in the case of any given society, that any of the average tables would represent its experience to the end of life. It is at these extreme ages that the sickness is most difficult to distinguish from mere superannuation, and the rate of claim will depend almost entirely on the management of the

society. The safest plan, therefore, appears to be that which is generally followed by the most experienced actuaries, of assuming that in extreme old age sickness will become practically chronic and must be treated as a pension at the lowest rate of pay. The age to be adopted must of course depend upon the nature of the society's experience, but it should in no case be taken later than age 80. Even where the experience of the society would appear to justify such a step, the actuary cannot be certain that that experience will remain uniform. The members now in the society at these old ages may happen to be, on the whole, in fairly comfortable circumstances, and may not make claims in every case of sickness; but it by no means follows that when the present younger members reach the same ages a similar state of things will exist.

This difficulty in determining the rates of sickness at the older ages has led some actuaries to propose the abolition of sickness benefits throughout life, and the substitution of sickness allowance ceasing at age 65 or 70, to be then replaced by a pension at a fixed rate of pay. While this would be a great improvement from the point of view of the actuary, the proposed change does not appear to be popular among the friendly societies themselves, and there can be no objection to the sickness benefit continuing throughout life, provided it is always treated in the manner suggested.

With regard to the effect upon the value of the sickness benefit of the age at which the pension is assumed to commence, it will be sufficient to refer to the various tables appended to this Essay.

Valuation of Funeral Benefits.

With respect to the funeral allowances, those payable at the death of the member present no difficulty. They are usually accompanied by an allowance at the death of a member's wife, provided that she die during the lifetime of her husband. For the purpose of accurately valuing these, the ages of the wives should be known, but it is quite sufficient in ordinary practice to assume the wives of the same ages as the husbands, and subject to like rates of mortality. Clearly, all members must be treated as married. It will be generally found that the value of the allowances at the death of the wives of the members, as given by the formula $\overline{A}^{1}_{yx}$, or $\frac{1}{2}\overline{A}_{xx}$ when equal ages are assumed, may be taken at very nearly $\frac{3}{5}$ of the value of a similar allowance at the death of the member.

Widows' and Children's Allowances.

Funeral allowances are occasionally continued to the widows of deceased members on payment of a small contribution, commencing at the death of the member. As this contribution is always less than the premium required to provide for the benefit, the sum payable at the death of the wife must be treated as an absolute assurance, and the annual widows' contributions valued by the formula $(a_x - a_{xx})$.

In other societies payments are made on the death of the children of the members who die under a certain age. In this case it is obviously not sufficient to take into account the children actually existing; provision must also be made for those who may be born to existing members, and who may die during the lifetime of their fathers before attaining the stated age. To estimate the provision necessary for such additional liability, it is best to consider that at the birth of each child the society assumes a liability that may be symbolically expressed, *e.g.*, as ${}_{|12}A^{1}_{0:x} \times$ (amount of the funeral benefit); where x represents the age of the member at the birth of his child. The above may be taken for practical purposes at a constant value, using an average age for x. It will be quite near enough in general to assume an average rate of mortality of, say, 1 per-cent for the life x taking ${}_{|12}A^{1}_{0:x}$ as equal to $A^{1}_{0:\overline{12}|}$ at 1 per-cent higher rate of interest. The average number of children born annually to members of various ages being determined from the experience of the society, the benefit resolves itself into a variable annuity and may be readily valued.

Funeral Benefits in Branch Societies.

The societies dealt with so far have been varieties of the ordinary friendly society granting simply funeral and sickness benefits. It remains to be noticed how far the practical process of valuation needs to be modified to suit other types of societies.

In the case of a branch of an affiliated order in which the funeral benefits are re-assured with the district, since the society is primarily liable the funeral benefit must be valued in the usual manner, and, provided the district funeral fund is solvent, credit may be taken for its re-assurance of the death monies. The solvency of this fund is not always, however, easy to determine. The whole question of district levies is at present a burning one, and the maintenance of the district funds by means of an equal contribution per member from all the branches of the district, quite irrespective of their ages, is not only wrong in theory but increasingly inconvenient in practice. Especially is this so where the branches differ greatly as to the average age of

the members, in which case the younger branches may find themselves burdened with annual levies considerably in excess of the premiums necessary to insure the whole of their funeral benefits. Various methods have been proposed to remedy this inequality, but the only sound one is to fix a contribution for each member, graduated according to his age, which shall be sufficient to insure his funeral benefit; and to accumulate the surplus contributions as a reserve fund.

Funeral Levies. In the case of an ordinary friendly society, where the death benefits are partly met by levies upon the members, the value of these latter may be computed by the formulas I have demonstrated elsewhere (*J.I.A.*, xxvi, 478), as follows:

Value of a levy of 1 per member at each death in a society of n members

$$= \frac{n(n-1)}{2} \bar{A}_{xx}$$

Ditto at each death of a member's wife

$$\frac{n(n-1)}{3} \bar{A}_{xxx}$$

where x may be taken as the average age of the members of the society. Where the funeral benefit is alone assured and is provided for by means of levies, great care must be used as to the inclusion of negative values at the earlier ages, a point which will be dealt with presently.

Burial Societies. The large burial societies not involving considerations of sickness do not present many special features that it is important to consider here. In their general characteristics they resemble closely the industrial assurance companies. The children's assurances, which form so large a proportion of the business, may, owing to the diminished benefit at the younger ages when the mortality is heaviest, and its gradual increase as the rate of mortality diminishes, be treated as temporary assurances having no liability, provided that the valuer is satisfied that the contribution in each year of age is adequate to meet the sums assured at that age.

The mortality prevailing amongst the societies differs considerably, mainly from difference of locality, but in most of the larger societies it is higher than that of the population generally. Hence an average table cannot be employed. Mr. Sprague's analysis of the Royal Liver experience shows that the rate of

mortality is higher (and considerably so) among the newly-entered than among those of the same age who have been longer assured. This will be seen from an inspection of the following table, which gives the rates of mortality, as deduced by Mr. Sprague, from the Royal Liver experience, those of a comparatively small Lancashire society, and those of the English Life Table (3).

TABLE 15.—*Comparative Rates of Mortality by the undermentioned Standards.*

Ages	Royal Liver Experience				Over Darwen Society	English Life No. 3
	At Entry	Assured 5 Years	Assured 10 Years	Ultimate Rates		
20	1·12	1·04	·81	·81	·66	·84
30	1·44	1·34	1·12	1·10	1·00	1·03
40	2·00	1·88	1·76	1·40	1·28	1·29
50	2·50	2·67	2·67	2·32	2·00	1·75
60	4·39	4·76	4·84	4·79	4·00	3·07
70	8·34	8·84	8·90	8·92	10·10	6·33

The question of secessions in these societies will be dealt with under that heading.

Subsidized Societies. Subsidized societies present a special feature in respect of the annual grant available, either fixed in amount, as is usually the case, or varying with the membership of the society. Where the subsidy takes the form of a fixed annual sum independent of the number of members, it is usual to capitalize the payments, but in doing so it seems desirable that any probable increase in the membership of the society should be taken into account, as the subsidy will thereby be rendered relatively less effective. This may be conveniently done by capitalizing the sum at a higher rate of interest than that assumed in the valuation. (It is clear that an annual growth of the membership of a society enjoying a fixed subsidy is financially equivalent to an annual diminution of the subsidy.)

Societies of Females. Female societies necessarily differ widely from ordinary friendly societies. Their benefits almost invariably include allowances during confinements, in addition generally to funeral and sickness benefits. The confinement allowances may generally be safely valued on the basis of the society's experience. The benefit may be very conveniently and not inaccurately treated as a temporary annuity, ceasing, say, at age 45, the amount of which

may be taken as the average annual sum paid for confinements divided by the number of members below that age.

Rate of Interest. With respect to the rate of interest to be assumed at a valuation the utmost care is needed. The tendency to adopt a high rate is certainly one to be guarded against, and it is to be remembered that the actual return made by the society upon its funds is not in itself a sufficient guide. The nature and solidity of the investments must also be looked to, and it may generally be said that only when there is evidence that these are of really sound character, and, at the same time, are remunerative, will it be safe to adopt a rate above 3 or 3½ per-cent. Probably the great bulk of societies in this country cannot anticipate with any confidence a return of more than the lower of those rates.

It must be remembered that a change in the rate of interest employed in the valuation will produce a much greater relative effect in the case of a friendly society than in that of a life assurance society; for, in the latter, an increased rate of interest, while diminishing the estimated value of the liabilities, diminishes also that of the assets, by giving a smaller net premium. In a friendly society, where the gross premiums are almost invariably valued, this compensating effect does not exist.

Net Premium Valuations. In those relatively few cases, where the contributions have been calculated by an actuary, and where the society's experience has been well within the assumptions upon which they have been based, both as regards rates of sickness and rates of interest, the valuation may fairly be made upon the same basis as that upon which the contributions were originally computed. Indeed, this would be the natural and proper method to adopt, leaving the same margins for contingencies and future profits as were found desirable in the first instance. Such a valuation will correspond to what is usually termed a net premium valuation in insurance companies.

Deficiencies. Where, as the result of a valuation, a deficiency is shown to exist, it is of importance that suitable steps should be taken to place the finances of the society upon a sound basis. This may be done in various ways, either by increasing the contributions by a reduction in the benefits granted, or by a combination of both methods. The method employed, however, must be carefully considered with reference to the causes of the deficiency, so far as these are ascertainable. These will generally be found to be either inadequate contributions (usually in consequence of a uniform rate

having been charged for all ages at entry), excessive sickness rates (which is, of course, practically the same thing, since the adequacy of the contributions must be tested by the actual experience of the society), or insufficient provision for management expenses. The circumstances of societies vary so widely, that it is not possible to lay down any very precise rules as to the best methods of procedure in these cases; the principal point to be attended to being that the burden of the readjustments made should fall upon those members who are mainly responsible for the deficiency. Where, for instance, the deficiency has arisen from insufficiency of contributions, a uniform increase of these for all members, old and new, would obviously be very unfair and ineffective besides, seeing that it would create a mass of negative values at the younger ages, which would be almost wholly lost by the withdrawals that would immediately take place. It would be preferable, in the first instance, to increase each member's contribution to the sum which he should originally have paid (on the valuation basis), having regard to his age at entry. If a re-valuation is now made, the deficiency will be found to have been considerably reduced, and, in the case of a young society, may even have practically disappeared. The remaining deficiency may now be met by a further increase in the contributions of the older members, in proportion to the duration of membership, or by a corresponding reduction in the benefits.

Where the deficiency is large, and the society an old one, it will be found that no practicable increase to the contributions will meet the case, and it is then necessary to have recourse to a reduction in the benefits; and here the same principle of throwing the loss upon those members who have created the deficiency should be followed. Further reductions of the sickness allowance in prolonged illness often prove the simplest mode of dealing with such cases, as such reductions affect but slightly the recent entrants at the younger ages, operating mainly by reducing the heavy values of the benefit in old age. Where a deficiency has arisen in part from abnormally heavy sickness rates, a decrease in the allowances will usually be found to work in the direction of reducing the rates in the future, and is thus doubly effective financially. In some cases it unfortunately happens that the deficiency has become so unmanageable that the only course open to the actuary is to recommend that the society should take advantage of the Act and submit to a dissolution.

Surpluses.

In those cases in which a valuation shows a surplus of assets over liabilities, it will generally be found desirable to allow such surplus to remain, except in special cases. Where the surplus is merely nominal, that is to say, bears only a small proportion to the *gross liabilities* of the society, and might disappear upon a comparatively slight change in the basis of valuation, it should invariably be left untouched. In the few cases in which there is a substantial surplus brought out, where the valuation has been a stringent one, and comparison with previous investigations shows the surplus to be a growing one, it may be dealt with. The circumstances of individual societies vary so greatly that no very precise directions can well be given as to the best method of appropriating such surplus. Speaking generally, however, cash distributions are to be avoided, and perhaps the simplest and, in the end, most satisfactory method, is to employ the surplus in reducing the members' future contributions. The greatest care should be exercised in augmenting the sickness allowances, especially those payable in continued sickness, lest the rate of claim be thereby increased, and the assumptions upon which the valuation has been based should fail to work out in the future.

Financial Effect of Changes in the Rates of Sickness and Mortality.

Effect on the Value of the Sickness Benefits.

It will frequently happen that the same causes which produce a change in the rate of sickness also affect the mortality prevailing among the members of the society. The result of such a change in the rate of mortality is hardly less important than a change in the sickness rate, although it may be easily overlooked, as its effect is entirely indirect. In the first instance, indeed, a light rate of mortality appears to be beneficial to the society, since a less sum is annually paid on account of funeral benefits, and it is overlooked that in consequence of the larger number of members surviving to the older ages, the liability in respect of the future sickness is largely increased. Especially is this the case in respect of the pension after 70, 75, or 80, as the case may be, into which the allowance practically resolves itself at this period of life. It is therefore important, in considering the suitability of a table of sickness as the basis of a valuation, to have regard, not only to the sickness prevailing in the society, but also to the probable rate of mortality that may be expected to obtain in the future.

With a view of illustrating the relative effect upon the financial position of the society of a deviation from the standard tables in respect both of sickness and mortality, tables—which are appended—have been constructed on the assumptions that

The Mortality Rates are	The Sickness Rates are
Normal	Normal
Normal	20 per-cent above Standard
Normal	20 per-cent below Standard
20 per-cent above Standard	Normal
20 per-cent above Standard	20 per-cent above Standard
20 per-cent below Standard	Normal
20 per-cent below Standard	20 per-cent above Standard
20 per-cent below Standard	20 per-cent below Standard

These tables give the monetary values, reckoning interest at 3 per-cent, for the sickness benefits to ages 65, 70, 75, and 80, with a pension afterwards at the reduced rate of pay, and also the values of annuities and assurances, and are based upon the Manchester Unity, and Ancient Order of Foresters' experience.*

A reference to the figures in these tables will show at once the relative effects of changes in the rates of sickness or mortality, on the full value of the sickness benefit at various ages. The following shorter tables for three specimen ages will show very clearly the relative value of the benefit under the varying conditions named, the value when both mortality and sickness are normal being taken at 1,000. It will be seen that when the reduction in the allowance is to half-pay only, the effect of the change in the mortality rates is relatively much greater, but that the effect of the variations in the sickness rates is somewhat less than when the reduction is to quarter-pay.

TABLE 16.—*Relative Value of Sickness Benefit (Reduction to Quarter-Pay and Pension after* 80)—*M.U.* 3 *per-cent.*

Mortality	Age 22			Age 42			Age 62		
	Sickness			Sickness			Sickness		
	Normal	+20%	−20%	Normal	+20%	−20%	Normal	+20%	−20%
Normal	1,000	1,190	810	1,000	1,185	815	1,000	1,170	830
+20 %	904	1,080	728	890	1,059	721	877	1,021	733
−20 %	1,120	1,328	912	1,145	1,350	940	1,164	1,340	988

* Four-figure logarithms were employed in the construction of these tables.

TABLE 17.—*Relative Value of Sickness Benefit (Reduction to Half-Pay and Pension after* 80)—*M.U.* 3 *per-cent.*

Mortality	Age 22			Age 42			Age 62		
	Sickness			Sickness			Sickness		
	Normal	+20%	−20%	Normal	+20%	−20%	Normal	+20%	−20%
Normal	1,000	1,185	815	1,000	1,178	822	1,000	1,169	831
+20 %	887	1,056	718	870	1,031	709	858	1,003	713
−20 %	1,124	1,328	920	1,176	1,372	980	1,196	1,370	1,022

TABLE 18.—*Relative Value of Sickness Benefit (Reduction to Quarter-Pay and Pension after* 80)—*A.O.F.* 3 *per-cent.*

Mortality	Age 22			Age 42			Age 62		
	Sickness			Sickness			Sickness		
	Normal	+20%	−20%	Normal	+20%	−20%	Normal	+20%	−20%
Normal	1,000	1,192	808	1,000	1,186	814	1,000	1,171	829
+20 %	904	1,081	727	882	1,051	713	860	1,002	718
−20 %	1,116	1,328	910	1,146	1,351	941	1,163	1,349	977

TABLE 19.—*Relative Value of Sickness Benefit (Reduction to Half-Pay and Pension after* 80)—*A.O.F.* 3 *per-cent.*

Mortality	Age 22			Age 42			Age 62		
	Sickness			Sickness			Sickness		
	Normal	+20%	−20%	Normal	+20%	−20%	Normal	+20%	−20%
Normal	1,000	1,186	814	1,000	1,179	821	1,000	1,160	840
+20 %	884	1,054	714	860	1,022	698	837	983	657
−20 %	1,151	1,357	945	1,180	1,379	981	1,197	1,372	1,022

Effect on the Net Liability.

The effect upon the sickness values of any change in the assumed rate of mortality will be partly compensated at a valuation by the effect on the value of the contributions and death benefits; hence these tables do not exhibit the net result of such variations in practice. With a view of showing more precisely how far the financial position

of an average Society would be affected by the supposed changes in the sickness and mortality rates respectively, the following table has been calculated. Upon certain assumptions as to ages at entry and rates of secession and mortality (based on the Ancient Order of Foresters' experience), a model society was constructed, with the following supposed benefits:

Sickness Allowance	. .	12*s.* per week 1st 6 months.
„ „	. .	6*s.* „ 2nd „
„ „	. .	3*s.* „ thereafter.
Funeral Allowance	. .	£10 at death of member.
„ „	. .	£5 „ member's wife.
Annual Contribution	. .	£1 per member.

The table shows the results of a valuation of societies of 20, 40, and 60 years' standing by the Manchester Unity Table, 3 per-cent interest, under the various conditions described.

TABLE 20.

Net Liability of a Society of 1,000 *Members—M.U.* 3 *per-cent.*

(Benefits and Contributions as described.)

Mortality	Age of Society 20 Years			Age of Society 40 Years			Age of Society 60 Years		
	Sickness			Sickness			Sickness		
	Normal	+ 20 %	− 20 %	Normal	+ 20 %	− 20 %	Normal	+ 20 %	− 20 %
Normal	15,745	21,078	10,412	23,434	29,182	17,686	26,321	32,081	20,561
+ 20 %	12,916	18,019	7,813	19,945	25,156	14,734	22,533	27,816	17,250
− 20 %	18,924	24,605	13,243	27,343	33,655	21,032	30,695	37,058	24,331

Low Mortality and high Sickness Rates combined.

A combination of a low rate of mortality and a high rate of sickness is by no means unusual, especially among rural societies, and in this case it is highly dangerous to ignore the effect of the former element. It may be of interest here to give an actual case where this combination of high sickness rate and low mortality occurred. The table below represents the experience of five years of a large country society; and it will be seen from an inspection of the figures that the sickness rates, especially in respect of reduced pay and at the more advanced periods of life, are much above the average amount, while the number of deaths in the society during the five years was considerably below the expected numbers, as estimated by either the Manchester Unity or the Ancient Order of Foresters' experience.

TABLE 21.—*Sickness and Mortality Experience of Essex Provident Society* (1876–80).

Ages	Numbers at Risk	Expected Sickness (in weeks)						Actual Sickness (in weeks)			Deaths		
		According to M.U. Tables			According to A.O.F. Tables						Expected		Actual
		Full Pay	Half-pay	Total	Full Pay	Half-pay	Total	Full Pay	Half-pay	Total	M.U.	A.O.F.	
15–24	610·5	411	30	441	481	29	510	297	28	325	4	4	3
25–34	2,598	1,952	348	2,300	2,047	351	2,398	1,446	997	2,443	21	22	19
35–44	5,411·5	4,908	1,454	6,362	5,316	1,568	6,884	3,650	3,433	7,083	60	65	53
45–54	6,553	8,470	4,539	13,009	8,880	4,475	13,355	5,781	8,712	14,493	112	125	91
55–64	7,737·5	16,164	14,130	30,294	15,195	14,706	29,901	10,079	26,252	36,331	240	260	181
65–74	3,681	12,865	21,392	34,257	12,073	23,281	35,354	7,850	46,125	53,975	228	248	196
75–84	958	3,845	13,935	17,780	4,192	14,149	18,341	1,217	32,358	33,575	120	122	118
85–94	46·5	134	1,169	1,303	243	949	1,192	11	1,739	1,750	11	11	10
All Ages	27,596	48,749	56,997	105,746	48,427	59,508	107,935	30,331	119,644	149,975	796	857	671

In this particular case the valuation of the society by the Manchester Unity Table without modification would have resulted in a net liability of £96,500. If, on the other hand, effect is given to the heavier rate of sickness found to prevail, the mortality still being assumed to be normal, the liability is increased to £114,500; while finally, a valuation based upon the actual rates of sickness and mortality observed during the five years brought out a net liability of £128,500. The difference between these last two items will show the extent to which the financial position of the society was affected by the light rate of mortality.

Secessions.

Secessions in an ordinary Friendly Society.

In the valuation of a Friendly Society it is usually assumed that the contract of the society by which it undertakes to pay certain benefits, in case of sickness or death, to the assured, in consideration of certain weekly or monthly contributions made by him, will be terminated only by the death of the member. In practice, however, many more members may leave the society by voluntary withdrawal than by death, and since these withdrawals are not provided for in the ordinary tables of the values of sickness and funeral benefits, it is important to consider their effect upon the finances of the society; the more so, that very erroneous views often exist upon this subject among those connected with friendly societies.

By the withdrawal of any member the society, on the one hand, is released from a liability in respect of the benefits assured to that member, and on the other hand it loses an asset in the shape of the present value of his future contributions. Whether, therefore, a loss or profit will result, will depend on the relative value of such future benefits and contributions. To take the most common instance of a friendly society charging the same contributions for all ages at entry: where such contribution is anywhere near sufficient to meet the benefits assured, it is clear that the younger entrants must pay at an unnecessarily high rate. In the cases of such members the present value of their future contributions at the time of entering the society will be greater than the value of the benefits assured to them; and this would continue to be the case until such members reached the age at which the average contribution was just sufficient to provide for the benefits. Until such time the withdrawal of any of these members would result in a loss to the society, but at those ages at which the value

of the benefits exceeded the value of the contributions, in other words, where there is a liability attaching to the assurance, the secession of members will in theory relieve the society of such liability.

Class of Lives Withdrawing.

It does not follow, however, that in practice such withdrawals will be of any great benefit. The members comprising a friendly society must necessarily differ greatly amongst themselves as to their relative health or liability to sickness; and although, for the purposes of valuation, each case is treated as an average one, that is to say, every member is considered to be in ordinary health, this will not actually be the case. It may be presumed that the healthier members of a society would be more likely to withdraw than those more liable to make sickness claims upon the fund; and statistics prove clearly that this is the case. In Mr. Neison's report on the mortality and sickness experience of the Ancient Order of Foresters, 1871 to 1875, a table is given showing the number of withdrawals amongst the members who during the five years made no claims for sickness, and amongst the members who made claims during the same period. The following abstract will show at various ages the proportion of members in either class withdrawing from the society; and it will be seen that the rate of withdrawals among those whom we may term the healthy members is very heavy, as compared with the rate prevailing amongst those who had made sickness claims.

TABLE 22.—*A.O.F. Experience* (1871–75).—*Rates of Secession per-cent.*

Ages	Amongst Members not Claiming Sickness Benefits during the 5 Years	Amongst Members Claiming Sickness Benefits during the 5 Years
20–24	15·50	3·28
25–29	11·03	2·76
30–34	7·66	1·99
35–39	5·20	1·40
40–44	3·06	·97
45–49	1·68	·68
50–54	1·03	·40
55–59	·52	·32

From this it would appear that the actual gain to the society's finances resulting from withdrawals is less in practice than calculation would show—one effect of the withdrawal of healthy

TABLE 23.—*Combined Secession Experience of Seven Societies.*

Year of Membership	Ages at Entry												Year of Membership
	Under 25			25 to 34			35 to 44			All Ages			
	At Risk	Withdrew	Rate per-cent	At Risk	Withdrew	Rate per-cent	At Risk	Withdrew	Rate per-cent	At Risk	Withdrew	Rate per-cent	
1	834	102	12·2	554	68	12·3	92	15	16·3	1,480	185	12·5	1
2	752	76	10·1	477	46	9·6	83	5	6·0	1,312	127	9·7	2
3	649	72	11·1	431	33	7·7	88	4	4·5	1,168	109	9·3	3
4	552	46	8·3	392	29	7·5	84	3	3·6	1,028	78	7·6	4
5	481	29	6·0	358	19	5·3	101	1	1·0	940	49	4·6	5
6–10	1,210	67	5·5	973	32	3·3	286	9	3·1	2,469	108	4·4	6–10
11–15	928	29	3·1	631	8	1·3	96	2	2·1	1,655	39	2·4	11–15
16–20	791	15	1·9	424	7	1·6	63	...	...	1,278	22	1·7	16–20
21–30	859	12	1·4	582	3	·5	41	...	...	1,482	15	1·0	21–30
31 and upw'ds	427	...	...	350	...	...	16	...	...	793	...	...	31 and upw'ds

members being to increase the average sickness rates prevailing in the society.

Classification as to Age and Membership.

In considering the extent to which a society's liabilities will be relieved by withdrawals, the age at which such withdrawals take place is a factor of considerable importance. It may be laid down as a general principle that the heaviest rates of withdrawal will be amongst the younger members of the society, and amongst those who have most recently joined. In other words, secessions on the whole occur mainly at the insuring ages where the members have been insured for a brief period only, and where, in consequence, the liability in respect of their insurances is comparatively small. In illustration of this point, I have prepared Table 23, based upon a somewhat limited experience, but sufficient to indicate approximately the law of secessions in an average friendly society.

[Table 23, *see preceding page.*]

If we adopt a somewhat broader grouping, as in the following smaller table, the law of the figures is still better brought out.

Table 24.—*Rates of Secession per-cent.*

Years Assured	Ages at Entry	
	Under 25	Over 25
1– 2	11·2	11·1
3– 5	8·7	7·7
6–10	5·5	3·3
11–20	2·6	1·4
21–30	1·4	·5
31 and over	...	...

Where a uniform contribution is payable by all the members, which, on the whole, is the most common case, the duration of the assurances in the case of those seceding will not be important except so far as it affects their ages. Where the contributions differ with the age it will not be sufficient to consider merely the ages at which withdrawals take place. Nearly all tables professing to show the effect of secessions have hitherto considered only this question of age, and not that of the duration of assurance, and the effect of this is greatly to exaggerate the effect of such secessions.

If we consider a group of members of a given age who have entered at various times, and consequently are paying various rates

of contribution, while such a group may be subject to an observed average rate of withdrawal, the actual rate prevailing amongst the different members of a group will differ very considerably. In the case of the members recently assured the rate will be very heavy, but the corresponding financial effect will not be in proportion to this high rate in consequence of the smallness of the liability of the society with reference to those members. The real relief from secessions will arise from amongst such members of the group who, having been assured for several years, and their assurances having thus attained a positive value, may nevertheless withdraw from the society. But the rate of withdrawal amongst this portion of the group will be very much below the average of the whole group. Consequently, to treat all the members at the given age as subject to one constant rate will considerably exaggerate the financial effect of such withdrawals. Hence it would be quite unsafe to employ tables similar to those of Mr. Neison and Mr. Ratcliffe based on the Ancient Order of Foresters and Manchester Unity experiences.

Effect of new Entrants at inadequate Contributions.

Another point of practical importance is the extent to which the effect of withdrawals is counteracted by that of new entrants. A very large proportion of societies unfortunately grant benefits out of proportion to the contributions received, and in these societies the value of such benefits is from the outset greater than the value of the contributions payable by the member; and every new member, although he may for some years add to the funds of the society, in reality increases the society's liability from the moment that he joins it. If, therefore, effect is given to the influence of withdrawals, it is equally important that account should be taken of the increased liability produced by new entrants.

I have made an estimate upon the basis of the Ancient Order of Foresters' rates of entry and secession (1871–75), and find that, assuming benefits of 12*s.* per week (with reduction to quarter-pay) in sickness, £10 at death of member, and £5 at death of wife, annual contribution £1, in an average lodge of 300 members, the net liability at a valuation being £2,271, the society's liabilities would during five years

Be diminished by secessions . .	£330
Increased by new entrants . .	454
Net loss	£124

Or about £25 per annum.

Secessions in Collecting Societies.

In ordinary friendly societies, secessions practically cease at about ages 55 to 60, but in some societies this is not the case. Notably in the large collecting societies, whether assurances for sick pay are granted or the business confined exclusively to death benefits, the effect of collecting weekly or monthly, as the case may be, the small contributions payable by the members is to create a large number of withdrawals at almost all ages throughout life. The experience of the largest of such societies, the Royal Liver, has been analyzed by Mr. Sprague, who has deduced from the records of the society for the five years, 1874–78, the following table, giving the adjusted rates of secession for various ages at entry and various durations of membership.

TABLE 25.—*Rates of Secession per-cent in Royal Liver Society, 1874–78 (Sprague).*

Years Assured	Age at Entry						
	10	20	30	40	50	60	70
0	22·91	23·87	22·61	21·22	19·22	14·95	11·71
5	5·97	6·36	5·44	3·83	3·06	2·77	2·02
10	3·96	3·50	2·66	1·85	1·45	1·60	1·11
15	3·80	2·83	1·39	1·21	1·17	1·14	...
20	3·41	1·89	1·20	1·18	1·16	1·11	...
30	1·89	1·20	1·18	1·16	1·11	...	...
40	1·20	1·18	1·16	1·11	...	...	...
50	1·18	1·16	1·11	...	...	...	...
60	1·16	1·11	...	...	...	...	...
70	1·11	...	...	...	...	...	...

This table shows that to the oldest age withdrawals take place; but that while, immediately after entry, the rate of withdrawal is extremely high, amounting at the younger ages to upwards of 20 per-cent in the first year, it rapidly diminishes with the duration of assurance, and finally, when such duration exceeds 15 or 20 years, it reaches a minimum value of about 1 per-cent. The financial effect of such a rate of withdrawal must be very considerable, when it is remembered that a constant rate of secession of 1 per-cent per annum at all ages is equivalent in its effect to an increase of 1 per-cent in the rate of interest. A practical advantage gained by a society from the heavy rates of secession during the first year of assurance should not be lost sight of. By means of the large proportion of the first year's contribution which is usually available for management

purposes, the fund by means of which the society is enabled to keep together its agency organization is largely augmented. There is also this distinction between a large collecting society and an ordinary friendly society or branch, that in the former case the rate of withdrawal will be a comparatively stable quantity. It will be a rate in the first place depending upon an average of an enormous number of cases spread over a large area, and subject to comparatively constant influences; and while circumstances may arise to considerably increase the rate of secession in such a society, either owing to disputes in the management or doubts as to its stability, there will always be, under the most favourable circumstances, a certain minimum rate below which the withdrawals will not fall. This can scarcely be said to be the case in a simple friendly society where, on the one hand, there is every inducement to the members to keep up their contributions, and, on the other hand, the numbers in the society can scarcely afford a sufficient basis upon which to estimate with any certainty the rates of withdrawal that may in future be expected. In such a society, while the rates of sickness and mortality may be comparatively stable, notwithstanding the smallness of its membership, the rate of withdrawal may be subject to very violent fluctuations.

Secessions in Subsidized Societies.

There is another class of society in which secessions stand on yet a different basis—namely, that class of subsidized societies, such as the large railway servants' funds, or the funds belonging to large firms or works where membership is compulsory. In these cases withdrawal from the employment of the firm generally means withdrawal from the society, and from the fact that during employment membership is compulsory, the rate of withdrawal at different ages really becomes a rate of retirement from employment, and does not depend so much upon the voluntary action of the members as upon circumstances which are beyond his control. In these cases it is clear that the rate at which the members withdraw at the different ages is a factor that may be ascertained with some degree of approximation, and also some amount of confidence as to the working out of a similar experience in the future.

How far to be allowed for in Valuations.

In considering the manner and extent to which the element of withdrawals should be taken account of at a valuation, effect must be given to all these various considerations. In the first place, the experience of the society must be sufficiently extensive to afford trustworthy

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data as to the rate of withdrawal that has prevailed: first, as to the various ages throughout life; secondly, as modified by the duration of assurance; thirdly, it must be clear from the nature and relation of the benefits to the contributions that the relief afforded to the society by the withdrawal of members is not practically destroyed by the introduction of fresh members, entailing fresh liabilities; fourthly, the experience of the society must have been sufficiently steady to give confidence in similar results in the future. It will generally be found that very few of these conditions are fulfilled in the case of an ordinary friendly society. As a rule the membership is much too limited to afford sufficient basis for the future calculation of such an uncertain quantity as the number of members seceding at different ages. Still less will it admit of further analysis in respect of duration of assurance, and consequently, whatever may be said in theory as to the propriety of taking account of secessions in the valuation of such a society, the practical difficulty will always remain—that there are no reliable data upon which their effect can be calculated. Where the membership of a society, however, is sufficiently large, and where the experience in respect of secessions has been steady, there can be no objection in principle to allowing for the effect of such secessions at the valuation. But, in order that this effect may not be greatly exaggerated, it is important that a more minute analysis should be made of the withdrawals than that of proceeding merely upon the basis of age. It is necessary that tables should be framed showing the various ages at entry, the rate of withdrawal that has prevailed at the different durations of the assurances, and the valuation of the liability to the society must be conducted on this basis, all members of given ages at entry being treated separately. In the majority of societies where the membership is large, it will be found that the effect of such secessions is not material, in consequence of their absolute cessation before the older ages are reached, and it is only in such societies as the burial and collecting societies that its effect becomes important.

Negative Values.

In conjunction with the question of secessions must be considered that of negative values. These arise, as has already been explained, mainly in such societies as have one uniform rate of contribution for all ages at entry, from the value of the contributions at certain ages exceeding that of the benefits. In these cases, the withdrawal of the member entails an absolute

loss upon the society, and this loss can only be effectually guarded against either by excluding such members entirely from the valuation, or simply by valuing their future contributions at a sum not exceeding the value of the benefits. In other words, these negative values must be excluded in the valuations, otherwise the insurances appear as assets in lieu of liabilities—assets which may be lost at any time by the voluntary withdrawal of the members in question. The cases in which it would appear to be safe to include a proportion of such negative values are, first, the large collecting societies where the vast numbers of members involved may make it possible to count with some degree of certainty upon a proportion of the negative values being actually realized; and, secondly, in those cases where membership of the society is compulsory, and where, consequently, the payment of the future contributions is assured. In order to eliminate all cases of negative values at a valuation, the safest plan is to calculate the contributions required to provide for the benefits at each age, and to examine the schedules prepared for the valuation, and in all cases where the contribution actually payable exceeds the calculated amount, to value only the latter.

Negative Values in Levy Societies.

There are two special cases which may present some difficulty, namely, levy societies and societies of females. As regards the former, where, as is usually the case, the levies are only a part of the contributions, the simplest method of excluding negative values is as follows: at the younger ages, where alone these values will appear, the value of the future levies to be paid by any given member may be taken as equal to $(n-1)A^1_{xy}$; those to be paid by the remaining members at his death $(n-1)A^1_{yx}$; together $=(n-1)A_{xy}$, where y is the age of the member, x the average age of remaining members, n the number in the society. This value will, of course, increase with the age of y; and making the reserve values (apart from the levies) equal at all ages to this minimum value, all negative values are necessarily excluded.

Where the levies are the sole contributions paid by the members, the following method must be adopted. Having made a valuation of the sums assured at each age in the usual manner, a table must be formed, showing, for each age,

Value of the sums assured at that age and upwards $=\Sigma A_x$
Number of members . . . . . $=n$
Average age of members . . . . . $=x$

If the amount of the levy be κ per member, the net liability in respect of the members at age y and over will then be

$$\Sigma A_x - \kappa \cdot \frac{n(n-1)}{2} A_{xy},$$

and by selecting the age y, so that the highest value of this expression may be obtained, all negative values will be *ipso facto* excluded, but not otherwise.

In Female Societies. In the case of female societies, the negative values do not necessarily appear at the youngest ages. In such a society, for instance, there may be a funeral allowance at the death of the member, the husband, and children, besides allowances at confinements. The liability in respect of the member's and husband's deaths is at first small, but increases with the age of the member; on the other hand, the prospective liability in respect of confinements and children's deaths is heaviest at the younger ages. In consequence of this feature, the contributions at the younger ages may be too small—in fact, not covering the current risk—but more than sufficient in middle life (say from 35 to 55), giving, at these ages, negative values for the liability. In order to effectually exclude all negative values from the valuation of a society of this description, it is not enough to strike out all that appear upon the face of the valuation, the *prospective negative values* must also be taken account of. This may be best done by treating the contracts at the younger ages (up to age 35 in the case supposed) as temporary assurances, ceasing at the age at which the negative values first appear.

Fluctuations in Death and Sickness Rates.

Theory of Fluctuations. The calculations of the actuary are all made on the assumption that certain average results will work out; and such expected results, being based upon those already observed during a series of years, it is important to know how far the assumption of a like experience in the future may be trusted. Three main causes tend to produce fluctuations from year to year in the sickness or mortality experience of a society: 1. Exceptionally healthy or unhealthy years, affecting all societies alike. 2. Changes in the management of the society, involving greater strictness or laxity in the conduct of the business. 3. The limited number of the members under observation, producing purely accidental fluctuations. Of these causes, the first is unimportant, the second is not susceptible of calculation, the

third being the only one of which the actuary can take cognizance. Even in the largest societies there will be fluctuations from the average experience of an accidental nature, and these fluctuations will, as a matter of fact, be larger in actual magnitude than in a smaller society. The idea that in a very large number of cases individual fluctuations are completely balanced, and so entirely destroyed, is quite erroneous. The actual deviations from the most probable result will be greater the larger the number of cases, but, on the other hand, the proportion which such deviations bear to the average or expected result is smaller. The theory of such deviations has been discussed at length by writers on probability, and where simple cases of probabilities are concerned is well known.

Application to Mortality Experience.

It may be shown that if n lives are exposed to risk for a year, the probability of death being in each case q, the average or "expected" number of deaths being consequently nq ($=m$, say), then, in a series of such observations, the average deviation from this mean result will be very nearly $\pm\frac{4}{5}\sqrt{nq(1-q)}$, which again (seeing that the factor $\sqrt{1-q}$ is nearly unity for all but the extreme ages) may be approximately written $\pm\frac{4}{5}\sqrt{\text{number of expected deaths}}$.* This result we may conveniently call the "probable error" of the observations, although this phrase is already used by mathematicians with a somewhat different meaning. It follows from this relation that if we have a series of observations at various ages, in which the expected deaths are respectively m_1, m_2, m_3, . . . m_n, and the probable errors a_1, a_2, a_3, . . . a_n (where $a_r=\frac{4}{5}\sqrt{m_r}$), then the probable error, when the whole of the results are combined, will be $\sqrt{a_1^2+a_2^2+a_3^2 \ldots +a_n^2}$. This result agrees, as would be expected, with what is known as the "Theory of errors of observation."

If, now, we consider a table of mortality experience, showing at each age the number exposed to risk and the numbers dying, it is clear that the rates of mortality deduced from these numbers will be affected by the accidental errors of which we are speaking, and to which are due the irregularities of all unadjusted tables. These irregularities, however, may be reduced by some process of graduation, when the probable errors in the resulting rates will be correspondingly smaller; and if the formula of graduation be known, an expression can be found for the mean deviation of the

* See the demonstration I have given of the formula in *J.I.A.*, xxvii, 214.

adjusted and unadjusted results. For example, if the formula of graduation, given on page 33, be employed, we have the following expression for this deviation:

$$u_x - u'_x = \frac{1}{120}\{96u_x - 22(u_{x-1}+u_{x+1}) - 17(u_{x-2}-u_{x+2}) - 10(u_{x-3}+u_{x+3})$$
$$-4(u_{x-4}+u_{x+4}) + 2(u_{x-7}+u_{x-6}+u_{x+6}+u_{x+7}) + u_{x-8}+u_{x+8}\}$$

Now, if we suppose each of the unadjusted values of u_x in the above expression to be affected with the same probable error, $\pm a$, then the probable error of the successive terms on the right of the equation will be, respectively, $\frac{96}{120}a$, $\frac{22}{120}a$, $\frac{22}{120}a$, $\frac{17}{120}a$, &c. &c., and the probable error of their sum will be found by taking the square root of the sum of the squares of these quantities $= \frac{\sqrt{11012}}{120}a$, or $\frac{7}{8}a$ nearly. Hence, in a table adjusted by this formula, the theory shows us that the average difference between the adjusted and unadjusted number of deaths at each age should be

$$\frac{7}{8} \times \frac{4}{5} \sqrt{\text{adjusted number of deaths}}$$
$$= \cdot 7 \sqrt{\text{adjusted number of deaths.}}$$

If Mr. Woolhouse's formula had been used, the coefficient would be nearly identical, namely, ·706; and it will be found, generally, that, for all practicable formulas of adjustment, the differences of the adjusted and unadjusted deaths will be very nearly

$$\sqrt{\tfrac{1}{2}\text{ adjusted number of deaths.}}$$

Having arrived at this result from theory alone, we are now in a position to test it by an application to actual facts. This is done in the following table, which shows, for the three divisions (rural, town, and city) of the Foresters' experience (1871-75), the average deviations between the actual and adjusted deaths in various groups of ages, and the computed deviations according to the formula just given. It will be observed that the average difference in each of the three divisions is small, and when the whole is combined we have the following results:

Average of observed deviations = 6·85
„ computed „ = 6·90

TABLE 26.—*Showing the Average Deviations of Deaths from Adjusted Results.—A.O.F. Experience*, 1871–75.

Ages	Rural Districts				Town Districts				City Districts				Ages
	Mean No. at Risk at each Age	Mean No. of Deaths at each Age	Mean Deviation from Adjusted Results at each Age		Mean No. at Risk at each Age	Mean No. of Deaths at each Age	Mean Deviation from Adjusted Results at each Age		Mean No. at Risk at each Age	Mean No. of Deaths at each Age	Mean Deviation from Adjusted Results at each Age		
			Observed	By Theory			Observed	By Theory			Observed	By Theory	
20–24	14,713	99	11·2	7·0	10,504	77	5·4	6·2	11,105	95	6·0	6·9	20–24
25–29	17,512	111	10·6	7·5	15,212	104	9·4	7·2	19,436	165	6·1	9·1	25–29
30–34	15,572	123	6·8	7·8	15,051	135	8·2	8·2	29,857	199	12·8	10·0	30–34
35–39	12,209	109	4·2	7·4	11,954	121	2·0	7·8	17,446	224	4·4	10·6	35–39
40–44	8,837	92	9·4	6·8	9,025	116	10·0	7·6	13,194	191	11·0	9·8	40–44
45–49	5,779	72	6·4	6·0	5,750	95	5·8	6·9	8,132	159	11·6	8·9	45–49
50–54	3,556	62	8·0	5·6	3,442	65	6·4	5·7	4,603	110	9·0	7·4	50–54
55–59	2,197	55	1·4	5·2	2,016	61	5·8	5·5	2,515	84	8·8	6·5	55–59
60–64	1,397	46	4·0	4·8	1,239	48	3·0	4·9	1,422	61	7·4	5·5	60–64
65–69	896	46	3·4	4·8	711	42	2·2	4·6	714	48	4·8	4·9	65–69
	Average		6·54	6·29	...	...	5·82	6·46	...	...	8·19	7·94	

We may take it, therefore, that a comparison of theory and observation confirms the formula given at the commencement of this section, and that in any given society, if the expected (or average) annual deaths $=m$, the average deviation from this mean number will be about $\frac{4}{5}\sqrt{m}$, or $\frac{4}{5}\sqrt{5m}$ in each quinquennium. If it is required that the average fluctuations shall not exceed 10 per-cent of the deaths in each quinquennium, we must have

$$\frac{4}{5}\sqrt{5m} < \frac{5m}{10}$$

which gives $m > 12{\cdot}8$; that is to say, for a society's mortality experience to attain this degree of uniformity there must be at least an average of 64 deaths in the quinquennium. This may be taken as equivalent to a membership of about 1,000.

Application to Sickness Experience.

In the case of sickness, however, the question is more complicated. It would naturally be supposed that the fluctuations would be less as regards the sickness than in the case of deaths, from the fact that the attacks of sickness are much more numerous than the deaths. Experience proves that this view is correct. Numberless small societies of 50 or 60 members contrive to carry on their operations from year to year, with, on the whole, a fairly steady experience in respect of sickness claims, which, did they grant death benefits alone, would be in considerable and constant danger from the smallness of their numbers. It is to be noted that the sickness claims in a society are liable to causes of fluctuation which do not affect the mortality experience. For example, there are cases of imposition which it is very difficult to entirely eliminate, but which scarcely affect the mortality experience. There may be insufficient supervision maintained in cases of prolonged sickness, and it is mainly in respect of these cases that such small societies run into risk. This risk, moreover, in respect of heavy claims for superannuation, will be greater or less according to the extent to which the benefit is reduced under these circumstances. If, for example, a member can draw half-pay for an indefinite period of sickness, it is clear that the finances of the society will be affected by such prolonged claim to a much greater extent than where only quarter-pay is allowed—*e.g.*, a society of fifty members, contributing annually £1 to the funds and receiving a sickness benefit amounting to 10*s.* full pay, might, by the prolonged illness of a single member, be subject to a drain of £13

per annum—upwards of a quarter of the society's entire income. If, however, the sickness claim in such cases was reduced to quarter-pay, the annual loss would be only half the sum named. It is evidently important therefore, that in small societies adequate provision should be made for the reduction of the sickness claims whenever they become chronic. Where the total amount that can be drawn by any member is limited the danger arising from such cases is, of course, avoided.

The fluctuations arising in the sickness experience of societies cannot be dealt with theoretically in the same manner as mortality fluctuations, since it is not merely the probability of sickness that is involved, but also the varying durations of illness. It is necessary, therefore, to proceed in the inverse order and to employ the observed deviations between the adjusted and unadjusted sickness claims at various ages to obtain a general expression for such fluctuations. The following table exhibits the results for the same experience as that dealt with in the table of mortality fluctuations given above:

[Table 27, *see page* 83.]

An analysis of these results shows that the mean differences observed between the adjusted and unadjusted numbers are, on the average, nearly equal to $\frac{7}{4}\sqrt{\text{average number of weeks' sickness.}}$ Allowing for the effect of the graduation formula in the same manner as in the case of the mortality experience, this would correspond to a probable error in the number of weeks' sickness observed of $2\sqrt{\text{expected number of weeks' sickness.}}$ Thus, in a society experiencing an average of about 5,000 weeks' sickness each year the probable error or average annual fluctuation would be $2\sqrt{5{,}000}$ weeks $=140$ weeks nearly.

TABLE 27.—*Showing Mean Deviation of the Observed Sickness from Adjusted Results.—A.O.F. Experience*, 1871–75.

Ages	Rural Districts			Town Districts			City Districts			Ages
	Mean Number at Risk at each Age	Mean Number of Weeks' Sickness at each Age	Mean Deviation from Adjusted Number at each Age	Mean Number at Risk at each Age	Mean Number of Weeks' Sickness at each Age	Mean Deviation from Adjusted Number at each Age	Mean Number at Risk at each Age	Mean Number of Weeks' Sickness at each Age	Mean Deviation from Adjusted Number at each Age	
20–24	14,713	11,758	83	10,504	8,751	194	11,105	9,253	136	20–24
25–29	17,512	14,523	240	15,212	13,409	224	19,436	16,583	175	25–29
30–34	15,572	14,210	236	15,051	14,834	150	20,857	20,769	216	30–34
35–39	12,209	12,939	198	11,954	13,465	241	17,446	21,428	281	35–39
40–44	8,837	10,697	96	9,025	12,268	334	13,194	19,677	171	40–44
45–49	5,779	8,459	107	5,750	9,888	127	8,132	15,205	253	45–49
50–54	3,556	6,325	230	3,442	8,041	160	4,603	11,915	234	50–54
55–59	2,197	5,989	65	2,016	6,370	203	2,515	9,235	251	55–59
60–64	1,397	5,589	206	1,239	6,051	94	1,422	7,091	139	60–64
65–69	896	6,501	177	711	6,494	116	714	5,505	145	65–69

The following table gives an illustration of the differences between the actual sickness claims during five successive years, in a large society of 5,500 members, and the estimated claims based upon the adjusted rates deduced from the society's experience for the period.

TABLE 28.—*Actual and Estimated Sickness.*

Year	Actual Weeks' Sickness	Estimated Weeks' Sickness	Deviation from Estimated or Average Amount
1876	17,694	17,487	207
1877	17,310	17,723	413
1878	17,580	18,002	422
1879	18,795	18,490	305
1880	18,774	18,943	169
Average	18,031	18,109	284

As computed by the above formula, the average deviation should be $2\sqrt{18109}=269$, very nearly in accordance with the observed amount.

To come to still smaller bodies, a society of 100 members, having say an annual average of about 120 weeks' sickness, may expect annual fluctuations to the extent, on the average, of about $\frac{1}{6}$th of the annual claims; or in a period of five years about 45 weeks out of a total of 600 weeks; hence, in such a society a "margin" equal to about 10 per-cent of the contributions would probably be sufficient to cover all contingencies. To produce an experience as steady as this, a society granting death benefits alone would require a membership of about 1,000.

Sickness and Death Benefits combined.

Where sickness and death benefits are combined in the same society the danger from fluctuations is lessened, for the reason that the financial effect of each death that takes place is to release the society from a liability in respect of future sickness claims, which will generally be greater than that incurred in respect of the funeral allowance. Hence, the plan of making a group of lodges and courts interdependent as regards their death payments, although from some points of view desirable, does not rest upon any actuarial basis, and does not

give additional stability to the individual societies. Theory goes to show that even small sickness societies of from 50 to 100 members may (always assuming careful management) attain a condition of permanency, and practical experience largely confirms this view.

As regards the present position and prospects of friendly societies, the principle of affiliation, which is daily becoming more popular, must have a beneficial influence upon the system as a whole, by fostering a spirit of emulation in the various orders, diffusing information, and affording the officials of the various branches opportunities for acquiring a wider experience, and by encouraging judicious supervision, especially in the matter of valuations. The benefits that accrue to the affiliated societies in these respects (in spite of various defects in the system still to be overcome), and the gradual improvement in their position, must, by the force of competition, spread to other societies, which must otherwise in the struggle for existence go down before their more vigorous rivals. Seeing that this is the natural tendency of things at this moment, and that the Act of 1875, as now administered, is gradually achieving what it was intended to do, namely, the improvement of the financial position of the societies, it would appear to be undesirable to attempt any drastic alteration in the principles upon which existing legislation is based.

The basis of the Acts regulating life assurance companies is publicity. Each company must show the public clearly what its transactions have been, in what manner and upon what assumptions its valuation estimates have been based, and what is the result of those estimates; and it is very doubtful whether any better principle can be applied to friendly societies.

TABLE 29.—*M.U. Experience—Rate of Mortality Normal—Rate of Sickness Normal—3 per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Values of Sickness Benefit (£1 per week Full Pay)								Age x
	$\bar{a}_x$	$\overline{A}_x$	Values of Annuity of £1, commencing at Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	21·90	·353	1·227	·654	·297	·110	57·76	46·33	40·27	37·77	39·09	34·31	32·00	31·19	22
27	20·80	·385	1·472	·784	·357	·132	65·16	51·43	44·19	41·11	42·92	37·18	34·43	33·38	27
32	19·57	·422	1·775	·945	·430	·159	74·05	57·49	48·76	45·06	47·48	40·56	37·23	35·97	32
37	18·21	·462	2·152	1·146	·521	·192	84·65	64·58	53·97	49·46	52·78	44·39	40·34	38·81	37
42	16·75	·505	2·634	1·403	·638	·235	97·74	73·18	60·40	54·68	59·19	48·93	44·08	42·10	42
47	15·16	·552	3·261	1·737	·790	·291	113·66	83·37	67·34	59·88	66·73	54·09	47·98	45·33	47
52	13·44	·603	4·103	2·186	·994	·367	133·79	95·54	75·32	66·72	75·84	59·86	52·15	49·22	52
57	11·66	·656	5·303	2·825	1·285	·474	160·22	110·77	84·66	73·54	87·18	66·51	56·57	52·76	57
62	9·86	·709	7·152	3·810	1·733	·639	197·71	127·13	95·81	80·79	102·39	74·57	61·09	55·96	62
67	8·14	·759	...	5·491	2·497	·900	211·10	160·29	109·51	87·35	105·56	84·75	65·39	57·73	67
72	6·51	·808	...	...	3·972	1·465	169·80	169·80	128·17	93·79	84·89	84·89	69·37	57·63	72
77	5·16	·848	...	...	...	2·757	133·64	133·64	133·64	103·17	66·82	66·82	66·82	56·83	77

TABLE 30.—*M.U. Experience.—Rate of Mortality Normal.—Values of Sick Benefit* (£1 *per week Full Pay*).—3 *per-cent.*

Age x	Rate of Sickness Increased 20 per-cent								Rate of Sickness Diminished 20 per-cent								Age x
	Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
	65	70	75	80	65	70	75	80	65	70	75	80	65	70	75	80	
22	62·91	52·20	46·78	44·75	43·72	39·47	37·43	37·14	52·59	40·46	33·76	30·79	34·46	29·15	26·57	25·24	22
27	70·54	57·64	51·17	48·65	47·68	42·58	40·39	39·71	59·78	45·22	37·21	33·57	38·16	31·78	28·47	27·05	27
32	79·63	64·07	56·28	53·25	52·36	46·21	43·56	42·75	68·47	50·91	41·24	36·87	42·60	34·91	30·90	29·19	32
37	90·39	71·54	62·06	58·35	57·74	50·29	47·05	46·17	78·91	57·62	45·88	40·57	47·82	38·49	33·63	31·45	37
42	103·59	80·52	69·16	64·39	64·18	55·07	51·24	49·91	91·89	65·84	51·64	44·97	54·20	42·79	36·92	34·29	42
47	119·43	91·01	76·70	70·46	71·60	60·39	55·52	53·70	107·89	75·73	57·98	49·30	61·86	47·79	40·44	36·96	47
52	139·21	103·28	85·22	78·16	80·34	66·15	60·00	58·11	128·37	87·80	65·42	55·28	71·34	53·57	44·30	40·33	52
57	164·69	118·23	94·91	85·78	90·83	72·47	64·54	62·08	155·75	103·31	74·41	61·30	83·53	60·55	48·60	43·44	57
62	200·06	133·54	105·96	93·63	104·27	79·58	68·80	65·49	195·36	120·72	85·66	67·95	100·51	69·56	53·38	46·43	62
67	211·10	163·79	118·43	100·14	105·56	87·42	71·98	66·94	211·10	156·79	100·59	74·56	105·56	82·08	58·80	48·52	67
72	169·80	169·80	133·15	104·93	84·89	84·89	72·92	65·35	169·80	169·80	123·19	82·65	84·89	84·89	65·82	49·91	72
77	133·64	133·64	133·64	109·47	66·82	66·82	66·82	61·03	133·64	133·64	133·64	96·87	66·82	66·82	66·82	52·63	77

TABLE 31.—*M.U. Experience.—Rate of Mortality Increased by* 20 *per-cent.—Rate of Sickness Normal.—*3 *per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Value of Sickness Benefit (£1 per week Full Pay)								Age x
	$\bar{a}_x$	$\bar{A}_x$	Value of Anny. of £1, commencing at Age				Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	21·03	·378	·964	·479	·197	·063	49·97	40·07	35·30	33·50	34·69	30·63	28·82	28·21	22
27	19·90	·412	1·164	·578	·238	·076	55·94	44·25	38·50	36·29	37·89	33·10	30·82	30·07	27
32	18·66	·449	1·415	·703	·289	·092	63·48	49·26	42·25	39·58	41·77	35·82	33·16	32·26	32
37	17·29	·489	1·731	·860	·353	·113	72·51	55·10	46·52	43·27	46·29	39·02	35·75	34·66	37
42	15·82	·532	2·143	1·064	·437	·140	83·83	62·27	51·66	47·64	51·84	42·85	38·81	37·46	42
47	14·23	·579	2·688	1·335	·548	·175	97·83	70·80	57·49	52·41	58·48	47·20	42·13	40·41	47
52	12·52	·630	3·440	1·708	·702	·224	115·80	81·16	64·16	57·64	66·57	52·11	45·66	43·45	52
57	10·76	·682	4·545	2·257	·927	·296	140·10	94·40	71·90	63·34	76·97	56·89	49·33	46·44	57
62	9·00	·734	6·321	3·139	1·289	·411	176·10	112·60	81·28	69·34	91·57	65·03	53·63	49·08	62
67	7·34	·783	...	4·731	1·943	·620	190·90	140·30	93·15	75·16	95·43	74·71	56·77	50·70	67
72	5·78	·829	...	...	3·301	1·053	150·30	150·30	110·30	79·72	75·15	75·15	60·34	49·99	72
77	4·51	·867	...	...	..	2·196	117·30	117·30	117·30	87·80	58·65	58·65	58·65	49·02	77

TABLE 32.—*M.U. Experience.—Rate of Mortality Increased by* 20 *per-cent.—Values of Sick Benefit* (£1 *per week Full Pay*).—3 *per-cent.*

Age x	Rate of Sickness Increased by 20 per-cent							
	Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age			
	65	70	75	80	65	70	75	80
22	54·71	45·59	41·34	39·87	39·12	35·51	34·07	33·69
27	61·08	50·09	44·96	43·15	42·44	38·10	36·37	35·89
32	68·82	55·46	49·20	47·02	46·44	41·16	39·04	38·47
37	78·01	61·65	53·99	51·34	51·05	44·59	41·98	41·30
42	89·45	69·19	59·72	56·44	56·64	45·65	45·44	44·59
47	103·42	78·02	66·14	61·98	63·19	53·17	49·13	48·04
52	121·10	88·51	73·34	68·00	70 94	58·09	52·96	51·56
57	144·50	101·54	81·46	74·47	80·55	62·60	56·79	54·96
62	178·50	118·80	90·83	81·07	93·45	69·88	60·90	57·83
67	190·90	143·80	101·68	86·97	95·43	77·35	63·07	59·23
72	150·30	150·30	115·20	90·19	75·15	75·15	63·83	57·25
77	117·30	117·30	117·30	93·94	58·65	58·65	58·65	53·11

TABLE 33.—*M.U. Experience.—Rate of Mortality Diminished by* 20 *per-cent.—Rate of Sickness Normal.—*3 *per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Value of Sickness Benefit (£1 per Week Full Pay)								Age x
	$\bar{a}_x$	$\overline{A}_x$	Value of Annuity of £1, commencing at Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter Pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	22·91	·323	1·583	·905	·456	·194	68·26	54·71	47·07	42·43	44·78	39·11	36·18	34·93	22
27	21·83	·355	1·886	1·078	·543	·231	77·22	61·06	51·95	47·64	49·40	42·63	39·14	37·66	27
32	20·64	·390	2·257	1·290	·649	·276	87·89	68·56	57·65	52·50	54·84	46·75	42·58	40·80	32
37	19·31	·429	2·712	1·550	·780	·332	100·05	76·29	64·17	57·98	61·14	50·93	46·40	44·27	37
42	17·86	·472	3·284	1·877	·945	·402	115·90	87·74	71·88	64·36	68·66	56·89	50·82	48·23	42
47	16·28	·519	4·012	2·293	1·155	·491	134·40	100·00	80·67	71·49	77·47	63·08	55·67	52·50	47
52	14·57	·569	4·966	2·838	1·429	·608	157·10	114·50	90·54	79·20	87·79	69·94	60·77	56·86	52
57	12·78	·622	6·278	3·589	1·807	·768	186·10	132·30	102·00	87·62	100·20	77·80	66·16	61·20	57
62	10·96	·676	8·212	4·694	2·363	1·005	225·40	155·00	115·40	96·59	116·20	86·79	71·61	65·15	62
67	9·05	·733	...	6·385	3·214	1·367	235·30	183·50	129·50	104·00	117·70	96·33	75·68	66·88	67
72	7·46	·780	...	...	4·857	2·065	194·00	194·00	147·90	113·00	96·98	96·98	77·51	67·86	72
77	6·01	·822	...	...	...	3·514	156·30	156·30	156·30	123·60	78·13	78·13	78·13	67·18	77

TABLE 34.

M.U. Experience.—Rate of Mortality Diminished by 20 per-cent.—Values of Sick Benefit (£1 per week Full Pay).—3 per-cent.

Age x	Rate of Sickness Diminished by 20 per-cent								Rate of Sickness Increased by 20 per-cent								Age x
	Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
	65	70	75	80	65	70	75	80	65	70	75	80	65	70	75	80	
22	62·84	48·47	40·03	34·75	39·94	33·64	30·13	28·45	73·68	60·95	54·11	50·11	49·62	44·58	42·23	41·41	22
27	71·58	54·45	44·38	39·21	44·42	36·91	32·72	30·73	82·86	67·67	59·52	55·97	54·38	48·35	45·56	44·59	27
32	82·85	61·56	49·49	43·44	49·74	40·75	35·75	33·36	93·73	75·56	65·81	61·56	59·94	52·75	49·41	48·24	32
37	94·50	69·09	55·39	48·11	55·96	44·77	39·15	36·28	106·50	83·49	72·95	67·85	66·32	57·09	53·65	52·26	37
42	109·80	79·95	62·42	53·58	63·47	50·39	43·11	39·63	122·00	95·53	81·34	75·14	73·85	63·39	58·53	56·83	42
47	128·40	91·92	70·54	59·75	72·41	56·93	47·54	43·28	140·40	108·08	90·80	83·23	82·53	70·23	63·80	61·72	47
52	151·50	106·35	79·86	66·52	83·15	63·33	52·33	47·07	162·70	122·65	101·22	91·88	92·43	76·55	69·21	66·65	52
57	181·50	124·50	91·00	74·09	96·47	71·58	57·63	50·96	190·70	140·10	113·00	101·15	103·93	84·02	74·69	71·44	57
62	223·00	148·41	104·62	82·50	114·30	81·64	63·43	54·73	227·80	165·59	126·18	110·68	118·10	91·94	79·79	75·57	62
67	235·30	180·01	120·31	90·31	117·70	93·66	68·90	57·06	235·30	186·99	138·69	117·69	117·70	99·00	82·46	76·70	67
72	194·00	194·00	142·84	101·14	96·98	96·98	73·90	59·66	194·00	194·00	152·96	124·86	96·98	96·98	81·12	76·01	72
77	156·30	156·30	156·30	107·15	78·13	78·13	78·13	62·88	156·30	156·30	156·30	130·05	78·13	78·13	78·13	71·48	77

TABLE 35.—*A.O.F.—Rate of Mortality Normal.—Rate of Sickness Normal.—3 per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Value of Sickness Benefit (£1 per week Full Pay)								Age x
	$\bar{a}_x$	$\bar{A}_x$	Value of Annuity of £1 commencing at Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	21·56	·363	1·117	·585	·263	·097	55·30	44·74	39·21	36·99	38·13	33·67	31·52	30·78	22
27	20·45	·396	1·344	·703	·317	·116	62·07	49·34	42·74	40·04	41·57	36·19	33·62	32·71	27
32	19·15	·434	1·620	·848	·382	·140	70·12	54·79	46·81	43·56	45·63	39·16	36·05	34·97	32
37	17·78	·474	1·972	1·032	·471	·170	79·87	61·21	51·67	47·53	50·42	42·55	38·84	37·43	37
42	16·30	·518	2·424	1·269	·572	·209	91·84	68·91	56·98	52·08	56·14	46·47	41·81	40·18	42
47	14·71	·565	3·021	1·581	·713	·261	106·72	78·12	63·27	57·19	62·95	50·88	45·09	43·05	47
52	13·02	·615	3·835	2·008	·905	·331	125·99	89·72	70·84	63·11	71·53	56·21	48·86	46·26	52
57	11·32	·665	5·032	2·634	1·187	·435	152·04	104·44	79·67	69·55	82·54	62·45	52·80	49·41	57
62	9·58	·717	6·875	3·598	1·622	·594	189·98	124·92	91·10	77·25	98·25	70·78	57·61	52·97	62
67	7·90	·767	...	5·273	2·376	·870	205·40	154·79	105·20	84·94	102·70	81·69	62·36	55·57	67
72	6·44	·810	...	...	3·917	1·431	167·44	167·44	126·05	92·47	83·72	83·72	67·99	56·69	72
77	5·12	·849	...	...	...	2·720	133·12	133·12	133·12	103·26	66·56	66·56	66·56	57·11	77

TABLE 36.—*A.O.F.—Rate of Mortality Normal.—Values of Sick Benefit* (£1 *per week Full Pay*).

Age x	Rate of Sickness Diminished by 20 per-cent								Rate of Sickness Increased by 20 per-cent								Age x
	Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
	65	70	75	80	65	70	75	80	65	70	75	80	65	70	75	80	
22	50·05	38·83	32·73	30·10	33·41	28·46	25·90	24·88	60·55	50·65	45·69	43·88	42·85	38·88	37·44	36·68	22
27	56·64	43·13	35·84	32·64	36·75	30·78	27·72	26·47	67·50	55·55	49·64	47·44	46·39	41·60	39·52	38·95	27
32	64·52	48·24	39·43	35·58	40·72	33·53	29·83	28·34	75·72	61·34	54·19	51·54	50·54	44·79	42·27	41·60	32
37	74·15	54·33	43·79	38·91	45·46	36·72	32·30	30·39	85·59	68·09	59·55	56·15	55·38	48·38	45·38	44·47	37
42	80·08	61·73	48·56	42·75	51·21	40·48	34·93	32·69	97·60	76·09	65·40	61·43	61·07	52·46	48·69	47·67	42
47	101·09	70·72	54·32	47·11	58·21	44·81	37·92	35·12	112·35	85·52	72·22	67·27	67·69	56·95	52·26	50·98	47
52	120·73	82·22	61·38	52·21	67·20	50·19	41·44	37·87	131·25	97·22	80·30	74·01	75·86	62·23	56·28	54·65	52
57	147·80	107·25	69·91	57·90	79·11	56·81	45·33	40·66	156·28	111·63	89·43	81·20	85·97	68·09	60·27	58·16	57
62	187·73	118·65	81·31	64·89	96·48	65·98	50·31	43·92	192·23	131·19	100·89	89·61	100·02	75·58	64·91	62·02	62
67	205·40	151·25	96·51	72·48	102·70	79·06	56·07	46·72	205·40	158·32	113·89	97·40	102·70	84·32	68·65	64·42	67
72	167·44	167·44	121·21	81·42	83·72	83·72	64·58	49·07	167·44	167·44	130·89	103·52	83·72	83·72	71·40	64·31	72
77	133·12	133·12	133·12	96·75	66·56	66·56	66·56	52·76	133·12	133·12	133·12	109·77	65·56	66·56	66·56	61·46	77

TABLE 37.—*A.O.F.—Rate of Mortality Increased by 20 per-cent.—Rate of Sickness Normal.—3 per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Value of Sickness Benefit (£1 per week Full Pay)								Age x
	$\bar{a}_x$	$\bar{A}_x$	Value of Annuity of £1, commencing at Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	20·67	·389	·863	·419	·170	·047	47·45	38·65	34·40	32·69	33·77	30·07	28·41	27·82	22
27	19·54	·423	1·046	·508	·206	·058	53·06	42·39	37·23	35·18	36·62	32·12	30·12	29·40	27
32	18·22	·462	1·271	·617	·251	·069	59·74	46·76	40·52	37·97	39·99	34·52	32·10	32·22	32
37	16·83	·503	1·562	·759	·308	·085	67·91	52·00	44·30	41·18	44·01	37·31	34·30	33·23	37
42	15·35	·546	1·944	·944	·383	·105	78·12	58·28	48·71	44·81	48·88	40·51	36·78	35·44	42
47	13·77	·593	2·458	1·193	·485	·133	90·97	65·88	53·81	48·86	54·73	44·15	39·47	37·73	47
52	12·09	·643	3·178	1·543	·627	·172	108·13	75·62	59·98	53·59	62·32	48·62	42·54	40·31	52
57	10·42	·692	4·276	2·076	·843	·231	131·89	88·26	67·21	58·61	72·32	53·92	45·74	42·73	57
62	8·71	·743	6·041	2·933	1·191	·327	168·20	106·55	76·83	64·68	87·33	61·33	49·78	45·54	62
67	7·09	·790	...	4·510	1·832	·502	184·34	134·73	88·98	70·31	92·17	71·60	53·81	47·30	67
72	5·70	·832	...	...	3·241	·889	148·20	148·20	108·05	74·93	74·10	74·10	58·90	47·32	72
77	4·47	·868	...	...	...	1·878	116·22	116·22	116·22	80·57	58·11	58·11	58·11	45·63	77

TABLE 38.—*A.O.F.—Rate of Mortality Increased by* 20 *per-cent.—Value of Sick Benefit* (£1 *per week Full Pay*).—3 *per-cent.*

Age x	Rate of Sickness Increased by 20 per-cent							
	Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age			
	65	70	75	80	65	70	75	80
22	52·45	44·20	40·40	38·98	38·28	34·99	33·65	33·26
27	58·23	48·23	43·60	41·91	41·22	37·22	35·61	35·13
32	65·07	52·90	47·32	45·21	44·68	39·82	37·87	38·26
37	73·37	58·45	51·56	48·97	48·75	42·80	40·36	39·65
42	83·64	65·03	56·46	53·23	53·60	46·16	43·14	42·25
47	96·38	72·85	62·05	57·94	59·29	49·88	46·10	44·93
52	113·23	82·72	68·72	63·41	66·52	54·33	49·42	47·92
57	136·03	95·12	76·27	69·13	75·67	59·31	52·70	50·68
62	170·42	112·61	86·00	75·92	89·09	65·97	56·64	53·80
67	184·34	138·22	97·25	81·76	92·17	74·19	59·81	55·45
72	148·20	148·20	112·81	85·29	74·10	74·10	62·25	54·47
77	116·22	116·22	116·22	86·92	58·11	58·11	58·11	49·87

TABLE 39.—*A.O.F.—Rate of Mortality Diminished by 20 per-cent.—Rate of Sickness Normal—3 per-cent.*

Age x	Annuity and Assurance Values (Continuous)						Value of Sickness Benefit (£1 per week Full Pay)								Age x
	$\bar{a}_x$	$\bar{A}_x$	Value of Annuity of £1, commencing at Age				Half-pay after 12 Months, and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
			65	70	75	80	65	70	75	80	65	70	75	80	
22	22·59	·332	1·466	·828	·369	·176	65·57	52·89	44·60	42·56	43·67	38·30	34·95	34·35	22
27	21·51	·364	1·751	·989	·440	·210	73·88	58·72	48·82	46·38	47·87	41·47	37·44	36·85	27
32	20·25	·402	2·054	1·183	·526	·251	82·62	65·56	53·70	50·79	52·27	45·14	40·32	39·61	32
37	18·90	·441	2·524	1·426	·635	·303	95·40	73·58	59·30	55·79	58·55	49·33	43·53	42·67	37
42	17·45	·484	3·067	1·733	·771	·368	109·62	83·11	65·74	61·47	65·34	54·14	47·07	46·03	42
47	15·87	·531	3·767	2·129	·947	·452	126·96	94·40	73·06	67·82	73·30	59·55	50·86	49·59	47
52	14·18	·581	4·696	2·653	1·180	·563	148·92	108·31	81·71	75·18	83·09	66·10	55·11	53·52	52
57	12·45	·632	6·009	3·395	1·510	·720	177·48	125·52	91·49	83·12	95·19	73·23	59·39	57·35	57
62	10·67	·685	7·940	4·487	1·996	·952	217·20	148·51	103·54	92·47	111·61	82·55	64·27	61·58	62
67	8·94	·736	...	6·268	2·215	1·330	232·44	180·90	103·17	102·63	116·22	94·79	61·81	65·50	67
72	7·39	·782	...	...	4·267	2·035	192·14	192·14	135·75	111·92	96·07	96·07	72·84	67·08	72
77	5·98	·823	...	...	...	3·491	155·48	155·48	155·48	124·13	77·74	77·74	77·74	67·67	77

TABLE 40.—*A.O.F.—Rate of Mortality Diminished by 20 per-cent.—3 per-cent.*

Age x	Rate of Sickness Diminished by 20 per-cent								Rate of Sickness Increased by 20 per-cent								Age x
	Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				Half-pay after 12 Months and Pension of Half-pay after Age				Half-pay after 6 Months, Quarter-pay after 12 Months, and Pension of Quarter-pay after Age				
	65	70	75	80	65	70	75	80	65	70	75	80	65	70	75	80	
22	60·08	46·62	37·60	34·96	38·75	32·79	28·92	28·02	71·06	59·16	51·60	50·16	48·59	43·81	40·98	40·88	22
27	68·21	52·12	41·34	38·20	42·85	35·75	31·10	30·03	79·55	65·32	56·30	54·56	52·89	47·19	43·78	43·67	27
32	76·78	58·60	45·70	41·94	47·16	39·19	33·62	32·34	88·46	72·52	61·70	59·64	57·38	51·09	47·02	46·88	32
37	89·44	66·28	50·74	46·21	53·40	43·17	36·48	34·92	101·36	80·88	67·86	65·37	63·70	55·49	50·58	50·42	37
42	103·64	75·50	56·60	51·09	60·25	47·82	39·66	37·78	115·60	90·72	74·88	71·85	70·43	60·46	54·48	54·28	42
47	121·16	86·59	63·37	56·61	68·43	53·18	43·15	40·85	132·76	102·21	82·75	79·03	78·17	65·92	58·57	58·33	47
52	143·56	100·44	71·50	63·07	78·68	59·78	47·16	44·28	154·28	116·18	91·92	87·29	87·50	72·42	63·06	62·76	52
57	173·26	118·07	81·04	70·24	91·78	67·41	51·44	47·75	181·73	132·97	101·94	96·00	98·60	79·05	67·34	66·95	57
62	215·05	142·14	93·21	78·93	109·93	77·75	56·61	51·74	219·35	154·88	113·87	106·01	113·29	87·35	71·93	71·42	62
67	232·44	177·31	94·05	89·02	116·22	92·13	55·21	55·86	232·44	184·49	127·21	116·24	116·22	97·45	68·41	75·14	67
72	192·14	192·14	13·79	100·12	96·07	96·07	69·37	58·96	192·14	192·14	140·71	123·72	96·07	96·07	76·31	75·20	72
77	155·48	155·48	155·48	117·46	77·74	77·74	77·74	63·21	155·48	155·48	155·48	130·80	77·74	77·74	77·74	72·13	77

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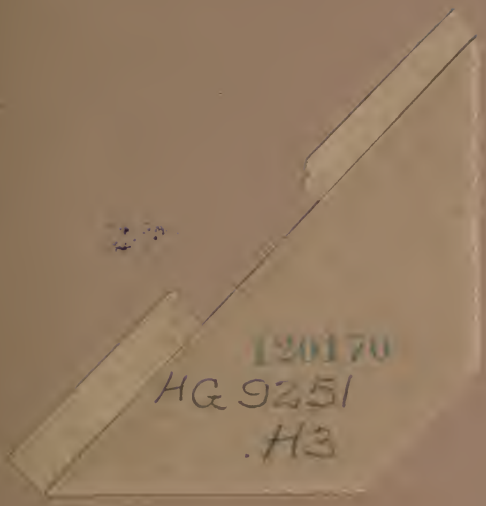
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